Meaningful Access To Grade-Level Mathematics for Students with Disabilities

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Exploring Research and Current Trends

Trends and Current Issues:

- Current OSEP Model Demo Grant on middle school mathematics
- Contemporary State Standards CCSS and implications for SWDs
- Regional Educational Lab Southeast: Math Alliance

Research:

- RtI Mathematics Practice Guide
- Research from NCSER Center on Learning of Fractions
- Current NSF research

Observations in Inclusion Middle School Mathematics Classes

- Often, teachers aware of need for teaching grade-level content to SWDs
- Using grade-level text is frustrating (too few examples, virtually no models, often paced too quickly for SWDs)
- Teachers rely on supplements with no empirical evidence (YouTube videos, Pinterest, Teachers Pay Teachers)
- No guidance on critical concepts and units to emphasize
- No guidance on what are foundational concepts and procedures necessary to support learning grade-level content
- New topics like statistics taught rather briefly

Emerging Trend: CCSS and Statistics

This is new for most mathematics teachers in K–12 and new for students

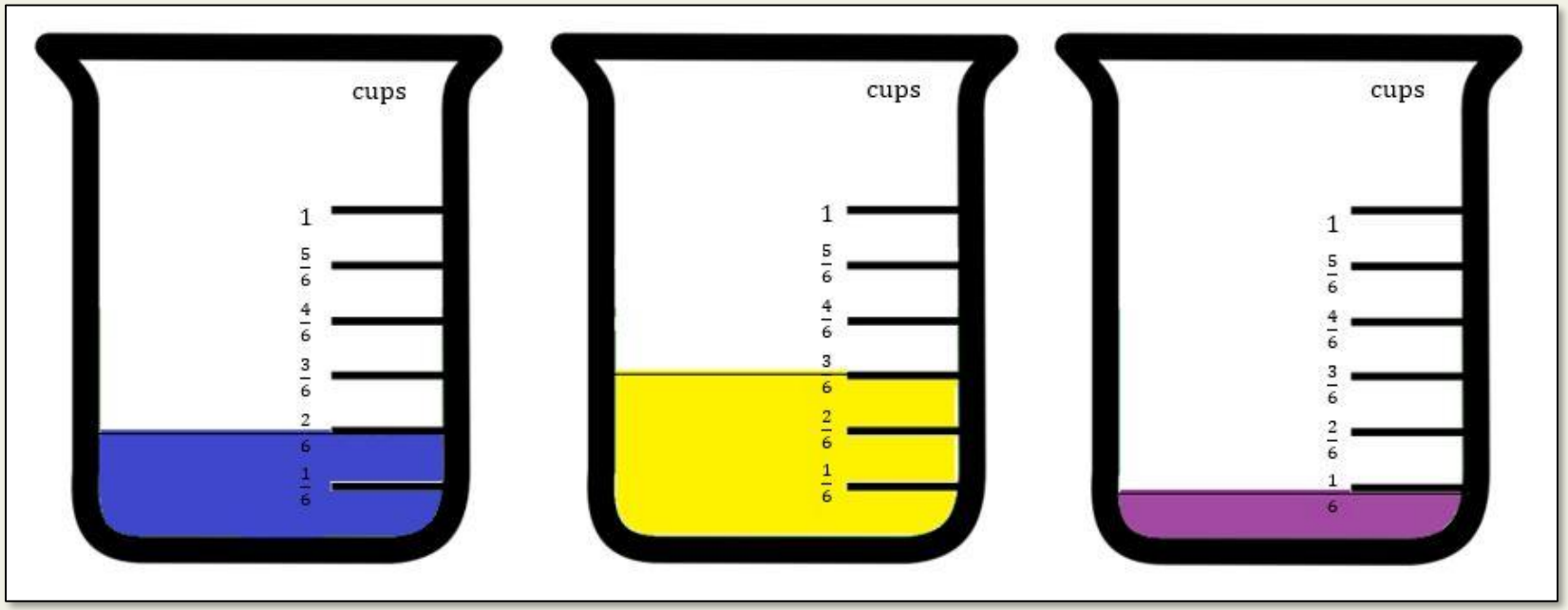
- Students with disabilities can be involved in this endeavor **from the onset**
- Would require a switch from remedial view of much of special education
- Believe it is achievable

Sample Statistics Standard from CCSS

- Represent and interpret data strand
- CCSS.MATH.CONTENT.5.MD.B.2
  Make a line plot to display a data set of measurements in fractions of a unit.
  - *Use…to solve problems involving information presented in line plots.*

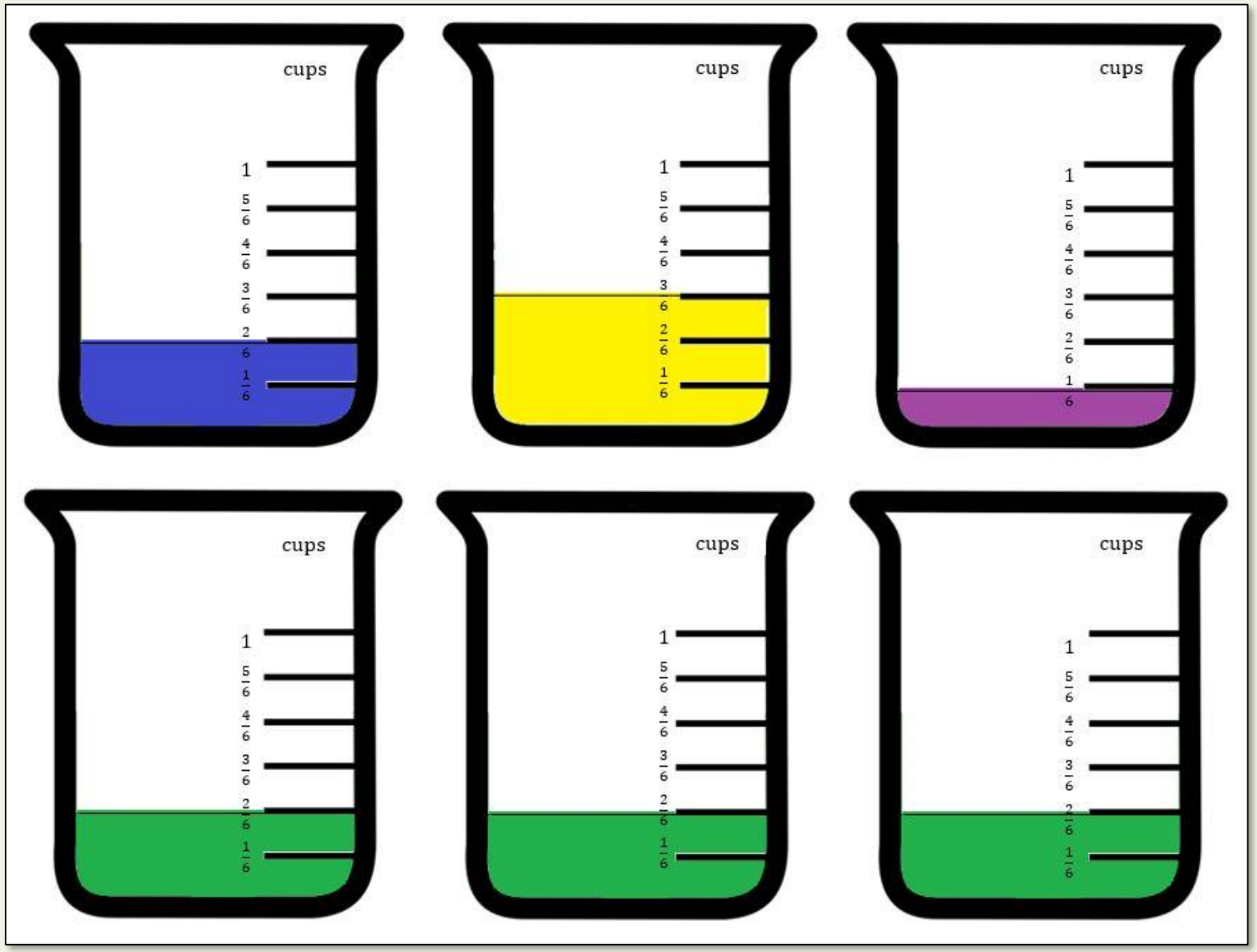
Sample Problem
Beaker Liquid Redistribution

Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.



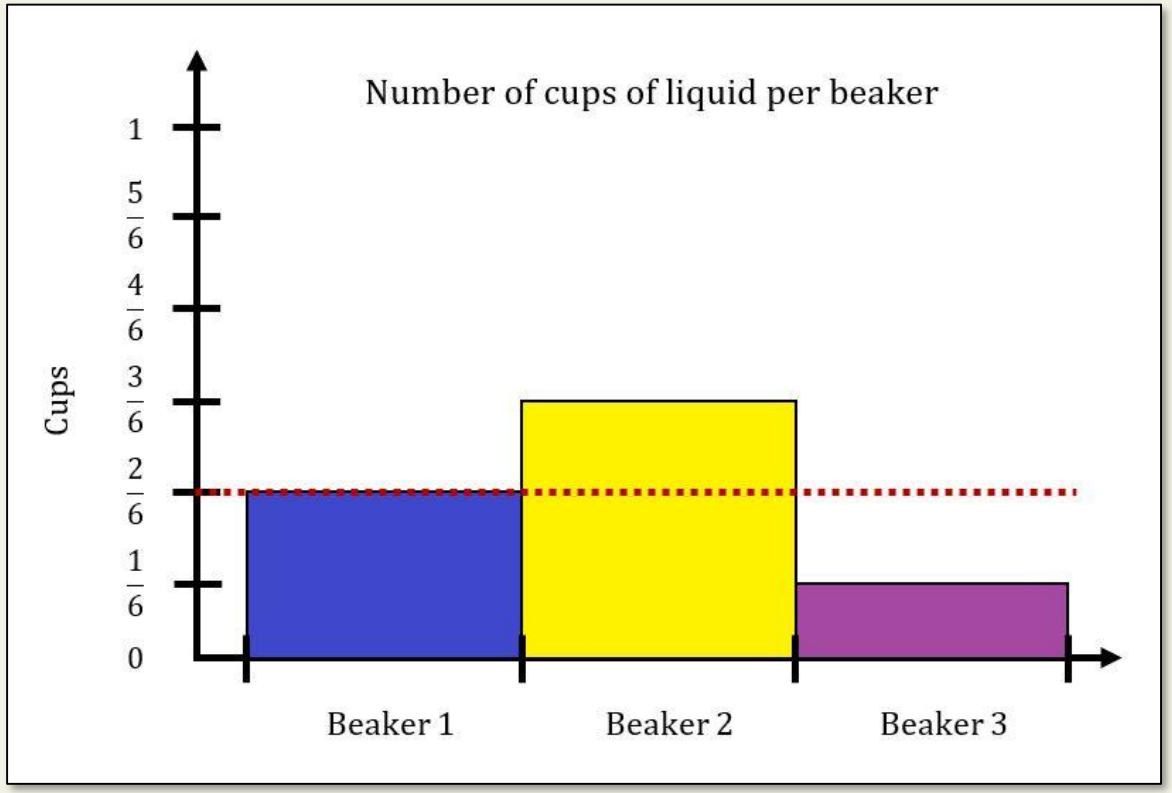
Original Distribution

"Fair Share" (or Mean) of Distribution



Increasingly Abstract Representations

- A very simple case of a distribution.
- Mean is $\frac{1}{3}$.
- Mean absolute deviation is $\frac{0 + \frac{1}{6} + \frac{1}{6}}{3} = \frac{1}{9}$.
- Moves on to larger distributions.
- Link between mean and fair share aspect of a fraction is made concrete.

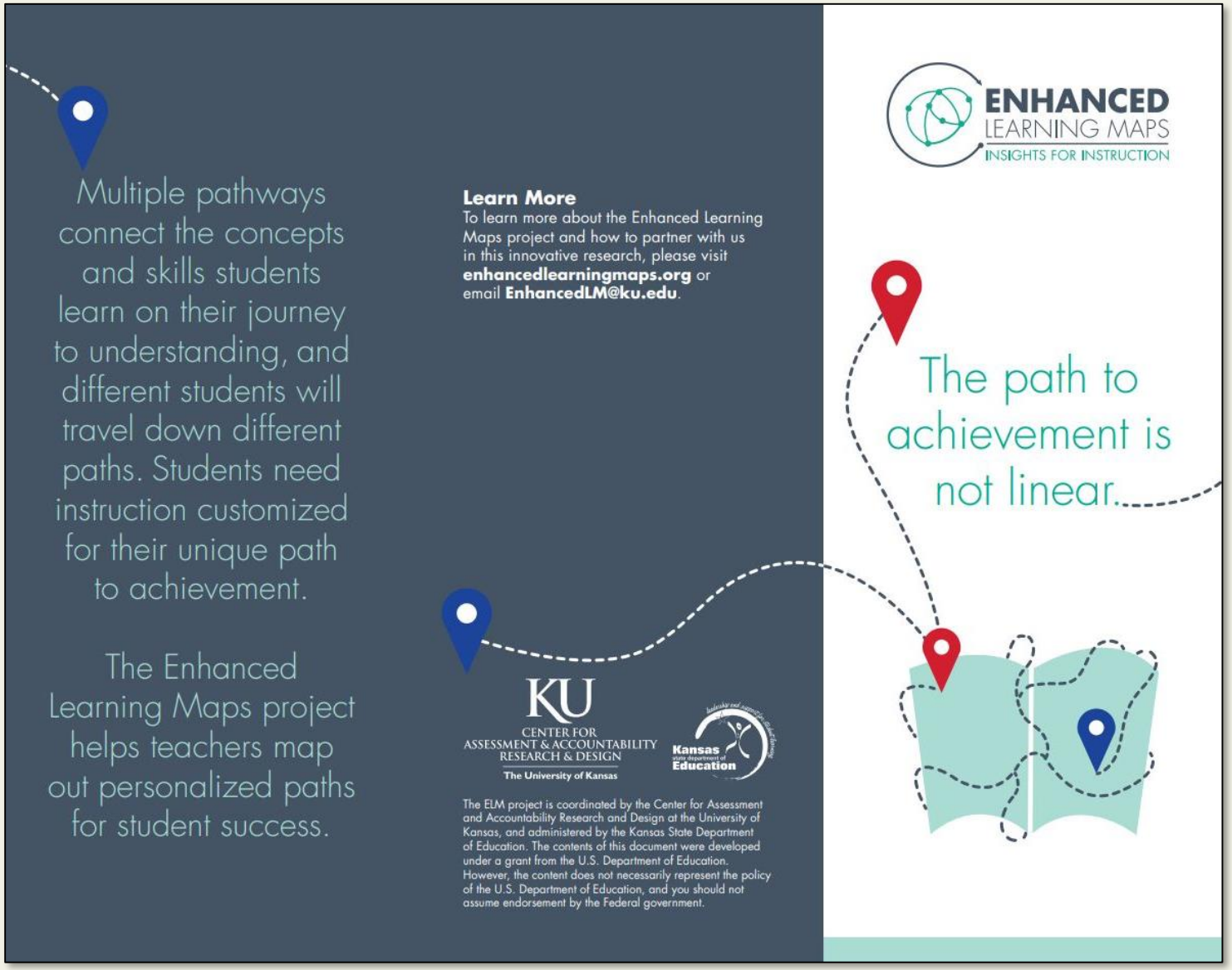


REL-SE MATHEMATICS ALLIANCE

- **Statistics** is a problem for all…many mathematics teachers even in high school don't know it.
  - Why not include special education teachers in PD for statistics?
  - Stats can be taught with less algebra, more focus on ideas
- **High algebra failure rate remains**…across SWDs and general population.
  - Need for both mathematics education and special education expertise as districts teach double dose, foundations of algebra…

Learning Progressions

- The Enhanced Learning Maps (ELM) project is a guiding framework.
- Formative assessment partnered with the rich information displayed in the learning map models
- Can guide teachers' instructional practice and approach to assessment.



Rational Number – Gateway to Algebra

- Knowledge of **fractions**, especially magnitude of fractions and **relative magnitude** of fractions is essential for success in algebra and beyond
- It is likely that this includes all aspects of rational number (decimals, proportions, rate and ratio, etc.)
- Fractions have been heavily researched in recent years
- IES Fraction Center, longitudinal and intervention focus
- **Our work** funded by NSF: RCT on 5th grade and meta-analysis of rational number intervention Grades 3–9

Fraction Magnitude: NAEP

- Many American students unable to solve fractions problems in middle school or even high school.
- Most think that the reason for poor performance on these items is that students never understood the mathematical ideas relating to fractions.

Solve It!

In which of the following are the three fractions arranged from least to greatest?

A. $\frac{5}{9}, \frac{1}{2}, \frac{2}{7}$

B. $\frac{5}{9}, \frac{2}{7}, \frac{1}{2}$

C. $\frac{1}{2}, \frac{2}{7}, \frac{5}{9}$

D. $\frac{1}{2}, \frac{5}{9}, \frac{2}{7}$

NAEP 2007 Grade 8: Pass rate = 49%

SWD Access Grade-Level with these Supports in Place

- Adequate time provided on **critical content**
- Explicit instruction and modeling
- Students are given tools to help them build and develop language of mathematics
- Adequate emphasis on ensuring students understand and can apply concepts, especially those with weak working memory (Fuchs, Schumacher, et al. 2013; 2014; 2016)

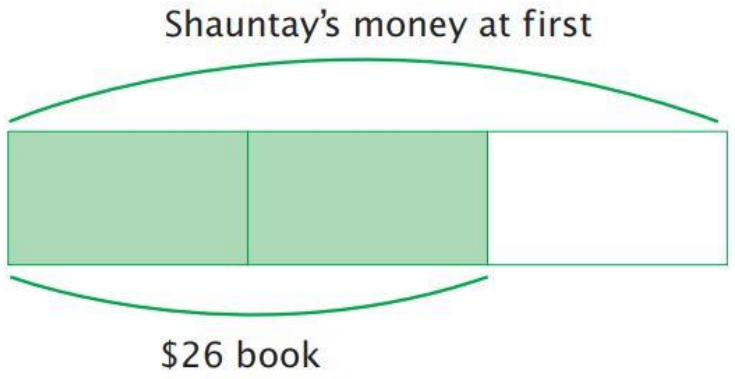
SWD Access Grade-Level with these Supports in Place

- Again, tools must be provided to promote concept development
- Peer interaction and frequent interaction with interventionist to reduce “learned helplessness”
- Use of visual representations (both concrete manipulatives and semi-concrete [e.g., strip diagrams, tape diagrams]) used in most Asian mathematics curriculum

STRIP DIAGRAM

Example 6. Strip diagrams can help students make sense of fractions

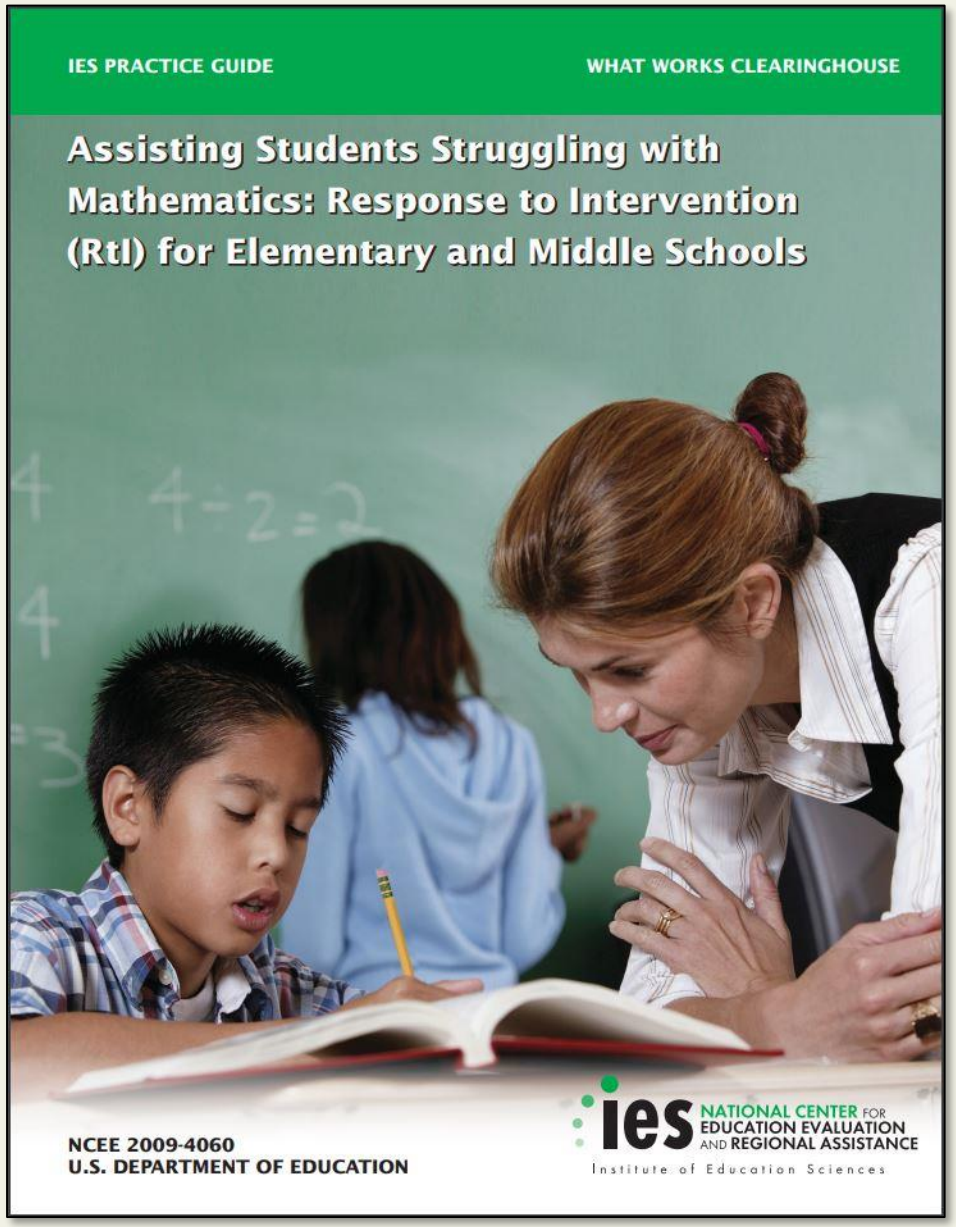
Shauntay spent $\frac{2}{3}$ of the money she had on a book that cost $26. How much money did Shauntay have before she bought the book?



2 parts → $26
1 part → $26 ÷ 2 = $13
3 parts → 3 × $13 = $39

Shauntay's had $39

IES Practice Guide: RtI in Mathematics



Our Work on 5th Grade Fractions Intervention: Enhanced TransMath

- National Science Foundation Grant 1535214
- Fifth-grade level, with fourth-grade foundations to support grade-level access
- CCSS emphasize student understanding and explanations – the research team enhanced the program with both oral and written opportunities for explaining
- Follows line of research of Fuchs, Schumacher, Malone, and colleagues (e.g., Fuchs et al. 2013; 2014; 2016)

5th Grade CCSS with Explanation Requirement

- CCSS.MATH.CONTENT.5.NF.B.5.B

  Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case)…and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.

Enhanced TransMath Addressed Student Explanations By:

- Providing additional opportunities for students to explain their work verbally
- Supporting students with written explanations of their problem solving
  - Prompt card with thinking and explaining steps
  - Supports students in thinking about the steps they took to solve the problem
  - Supports students in explaining why they may need to perform a specific step

SUPPORTING EXPLANATIONS

WRITING EXPLANATIONS FOR MATH PROBLEMS

THINKING

1. **What's the problem asking?**

Compare, use a number line, draw a picture? Add, subtract, multiply, divide?

2. **What did I do to solve it?**

Mark a number line? Use fair shares? Use benchmark fractions? Rewrite fractions to get common denominators?

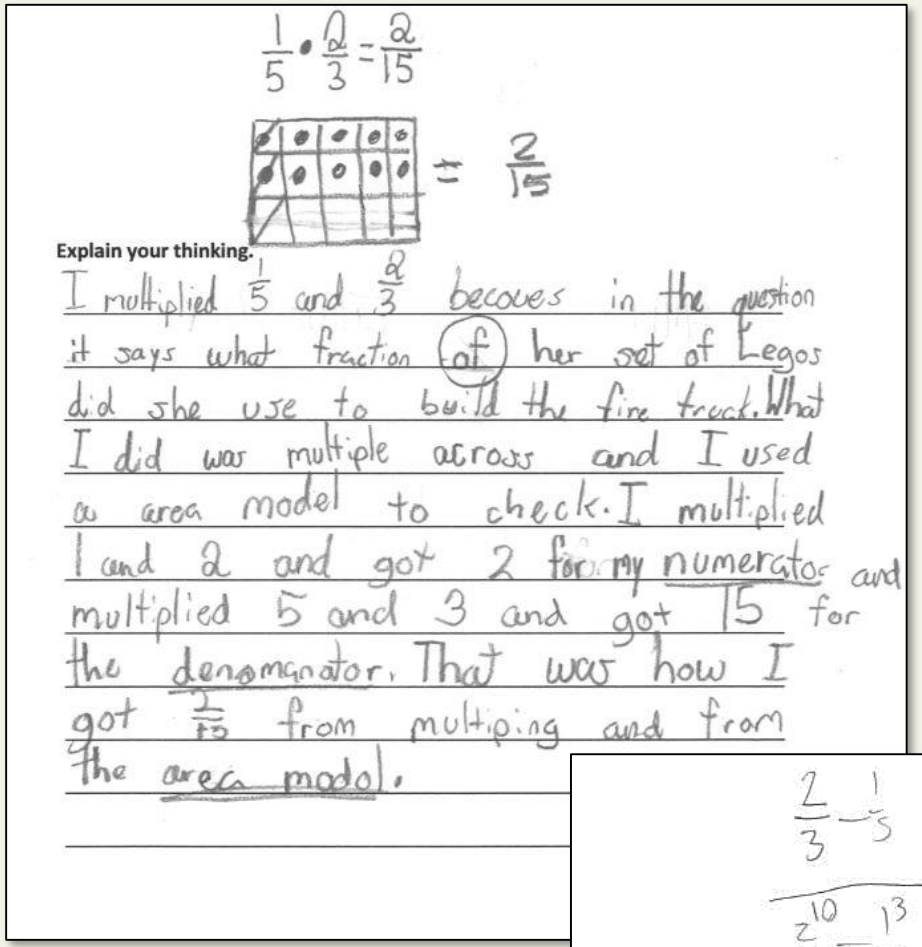
WRITING

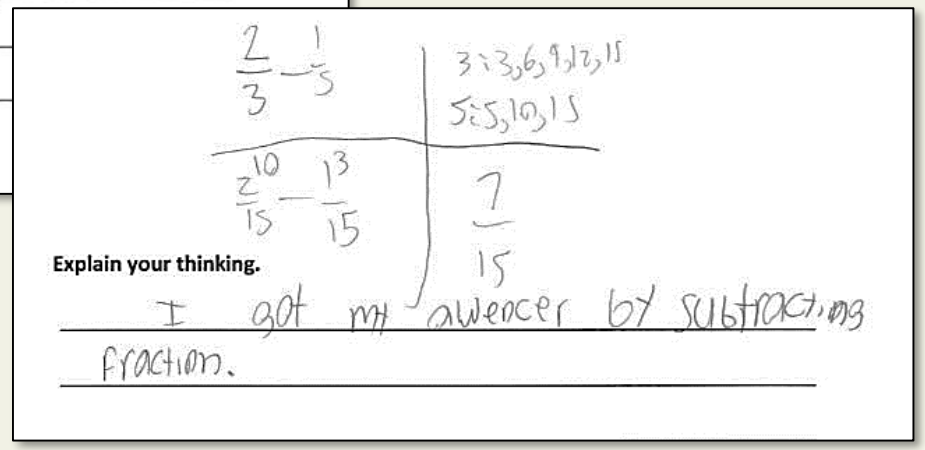
3. **Write all the steps.**
   - **Remember to use mathematically correct vocabulary.**

4. **Write the answer and say why it makes sense.**

Sample Explanations

Bella likes to build with Legos. In her set of Legos, $\frac{1}{5}$ are red. Bella used $\frac{2}{3}$ of her red Legos to build a fire truck. What fraction of her total set of Legos did she use to build the fire truck? Explain your thinking.





DATA FROM TRANSMATH STUDY

		TUF4	*TUF5*	*Fractions Procedure Test*	*NLE 0-1*
Fixed Effects	**Condition (*TransMath*)**	3.25 (.56)	2.36 (.46)	10.79 (1.37)	10.79 (1.20)
	WRAT4 Pre	.16 (.02)	.14 (.02)	.40 (.06)	.27 (.06)
	NLE 0-1 Pre	.14 (.02)	.09 (.02)	.31 (.05)	.37 (.05)
Hedges' *g*	**Condition**	**0.784**	**0.660**	**1.068**	**-1.083**
ICC	**Tutoring Groups**	.31	.13	.30	.11

ALL results significant at *p* = .0001.

Data from TransMath Study

- Performance Assessment Effects:
  - Accuracy: Hedges' g = .73 – 1.25
  - Explanations: Hedges' g = 1.04 – 1.12

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