It Isn't All about Algorithms and Facts: Mathematical Content for Learners Who Struggle

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Session Description

1. Overview of projects
2. Lessons learned
3. Discussion

Project Descriptions

- Algebra-readiness Intervention Modules
  - Funded by IES, Goal 2, #R324A120364
    - Diane Bryant, PI, Barbara Dougherty and Brian Bryant, Co-PIs
  - University of Texas-Austin, University of Missouri-Columbia
  - 7 modules to be used in middle school (grades 6–7) intervention classes

Project Descriptions

- Algebra Screening & Progress Monitoring Project
  - Funded by IES, Goal 5, #R324A110262
    - Anne Foegen, PI, Barbara Dougherty, Co-PI
  - Iowa State University, University of Missouri-Columbia
  - Development of conceptual screening and progress monitoring tools for Algebra I, high school

Beliefs about mathematical learning

- Mathematics is more than performing algorithms.
- The learning process should include opportunities to language about mathematics.
- Some topics can be taught concurrently, rather than sequentially.
- Understanding quantitative relationships is fundamental.
- Developing generalizations supports connections among concepts and skills and provides a means for longer retention.

Lessons learned

- Present tasks in a different way by changing how we ask questions
  - Questioning framework developed from work at CRDG with Krutetskii's studies

Questioning framework

Type of Question	Fractions	Integers
Standard type of question	$\frac{1}{2} \times \frac{3}{4}$	$-3 + -8$
Reversibility question	What are two fractions whose product is $\frac{3}{8}$?	What are two integers whose sum is -11?
Flexibility question	$\frac{1}{2} \times \frac{3}{4}$ $\frac{1}{2} \times \frac{2}{4}$ $\frac{1}{2} \times \frac{1}{4}$ How are these problems alike?	$-3 + (-8)$ $-4 + (-8)$ $-5 + (-8)$ How are these problems alike?
Generalization question	If the factors of a multiplication problem are between 0 and 1, what can you predict about the size of the product?	What are two negative integers whose sum is negative? What are a positive integer and a negative integer whose sum is negative? What are two positive integers whose sum is negative? What do you notice about the integers that you found?

Dougherty, B., Bryant, D. P., Bryant, B. R., Darrough, R. L., & Pfannenstiel, K. H. (2015). Developing concepts and generalizations to build algebraic thinking: The reversibility, flexibility, and generalization approach. *Intervention in School and Clinic*, 50(5), 273–281.

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Why does it matter?

Dan challenged Amy to write an equation that has a solution of 3. Which equation could Amy have written?

A. $4 - x = 10 - 3x$

B. $3 + x = -(x + 3)$

C. $-2x = 6$

D. $x + 2 = 3$

Dougherty, B J. & Foegen, A. (2016). *Conceptual progress monitoring for algebra.* Funded by IES, Goal 5, R324A110262

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Why does it matter?

Dan challenged Amy to write an equation that has a solution of 3. Which equation could Amy have written?

A. $4 - x = 10 - 3x$	(119/490; 24.3%)
B. $3 + x = -(x + 3)$	(135/490; 27.6%)
C. $-2x = 6$	(95/490; 19.4%)
D. $x + 2 = 3$	(141/490; 28.8%)

Dougherty, B J. & Foegen, A. (2016). *Conceptual progress monitoring for algebra.* Funded by IES, Goal 5, R324A110262

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Lessons learned

- Focus on generalizations that don’t ‘expire’
  - Use multiple tasks that lead to conjectures and then move to generalizations
  - Record conjectures and generalizations to archive thinking

Why does it matter?

Mari said, "2*t* is always greater than *t* + 2." Do you agree with Mari?

A. Yes, because multiplication always gives you a larger answer than addition.

B. Yes, because *t* is a positive number.

C. No, because multiplication is not the inverse of addition.

D. No, because it is possible that 2*t* can be equal to or less than *t* + 2.

Dougherty, B J. & Foegen, A. (2016). *Conceptual progress monitoring for algebra.* Funded by IES, Goal 5, R324A110262

Why does it matter?

Mari said, "2*t* is always greater than *t* + 2." Do you agree with Mari?

A. Yes, because multiplication always gives you a larger answer than addition. (312/750; 41.6%)

B. Yes, because *t* is a positive number. (64/750; 8.5%)

C. No, because multiplication is not the inverse of addition. (107/750; 14.3%)

D. No, because it is possible that 2*t* can be equal to or less than *t* + 2. (267/750; 35.5%)

Dougherty, B J. & Foegen, A. (2016). *Conceptual progress monitoring for algebra.* Funded by IES, Goal 5, R324A110262

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Lessons learned

Concrete–Representational–Abstract (CRA)

Concrete–Semi-concrete–Abstract (CSA)

Not linear or sequential: Should be presented simultaneously to form connections across the representations

INTERVENTION IN FRACTIONS AT 5TH GRADE: THE CENTRAL ROLE OF THE NUMBER LINE

Funded by NSF Grant DRL-1535214

STEM Education, Learning Disabilities, and the Science of Dyslexia
Washington, D.C.
September 26th, 2017

Madhavi Jayanthi
Robin Schumacher

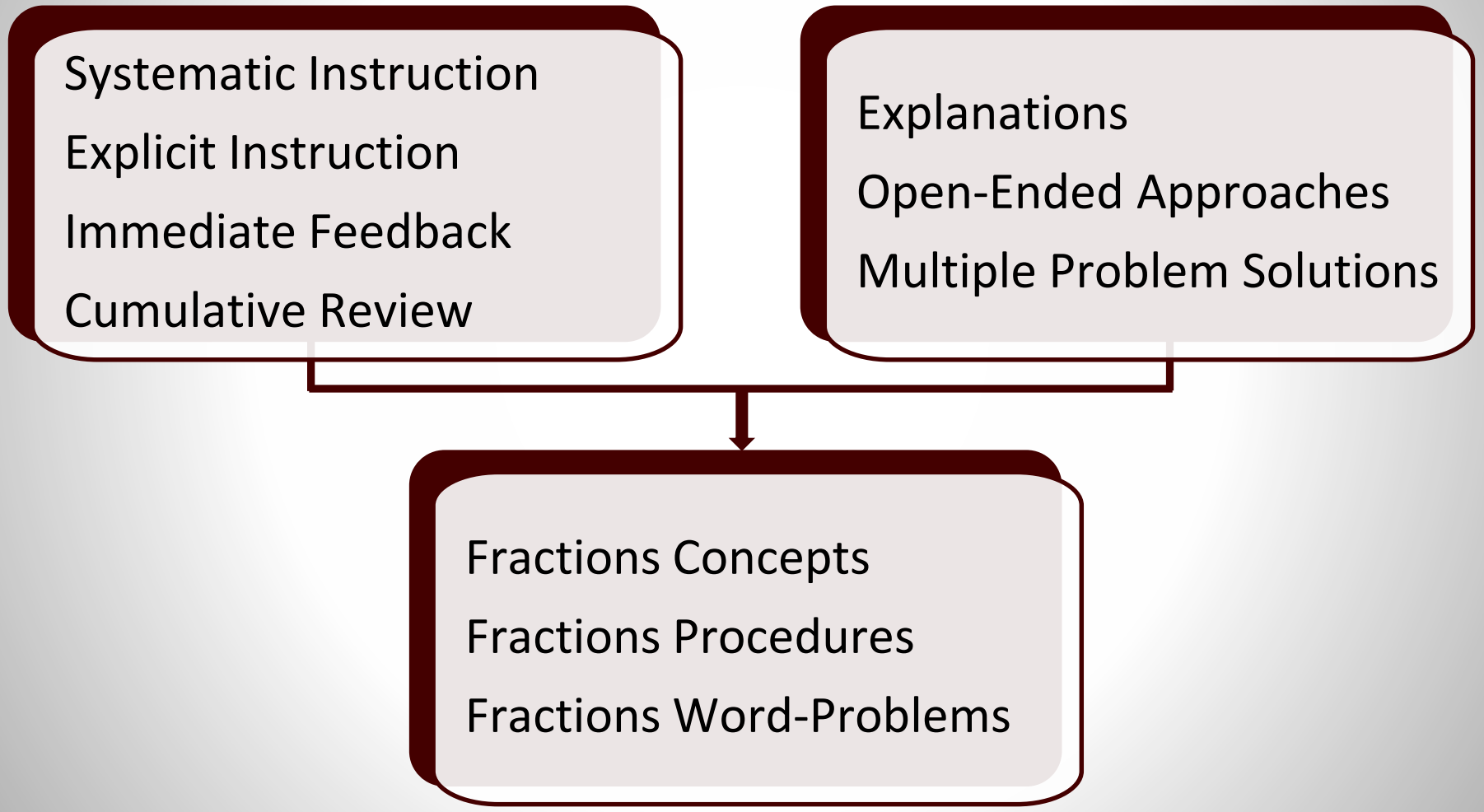
Instructional Research Group

GOALS FOR THIS SESSION

Describe the fractions intervention and draw attention to number line instruction.

Share findings from an RCT.

MERGING IDEAS FROM SPECIAL AND MATHEMATICS EDUCATION



FRACTIONS INTERVENTION

- *TransMath*® was adapted for small groups (Level 2; Woodward & Stroh, 2015)
- Linking part-whole to measurement understandings
- Concrete-Representational-Abstract
  1. Cuisenaire Rods
  2. Number Lines
  3. Equations

CENTRAL ROLE OF THE NUMBER LINE

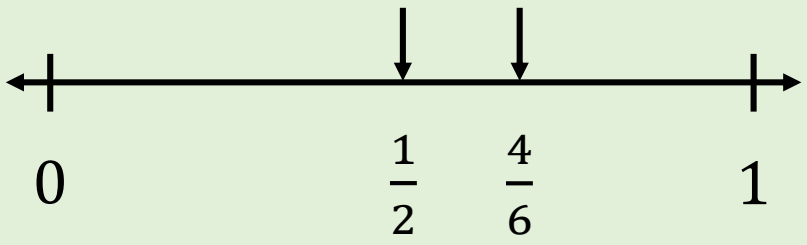
Superior representation for understanding magnitude

(Siegler et al., 2012)

- Consolidating rational number and whole number principles
- Equivalence of fractions
- Relative magnitude of fractions
- Four operations ($+ - \times \div$)

COMPARING FRACTIONS WITH BENCHMARK NUMBERS & RELATIVE SIZE

$\frac{4}{6}$ is here because it is $\frac{1}{6}$ greater than $\frac{3}{6}$, which is equivalent to $\frac{1}{2}$.



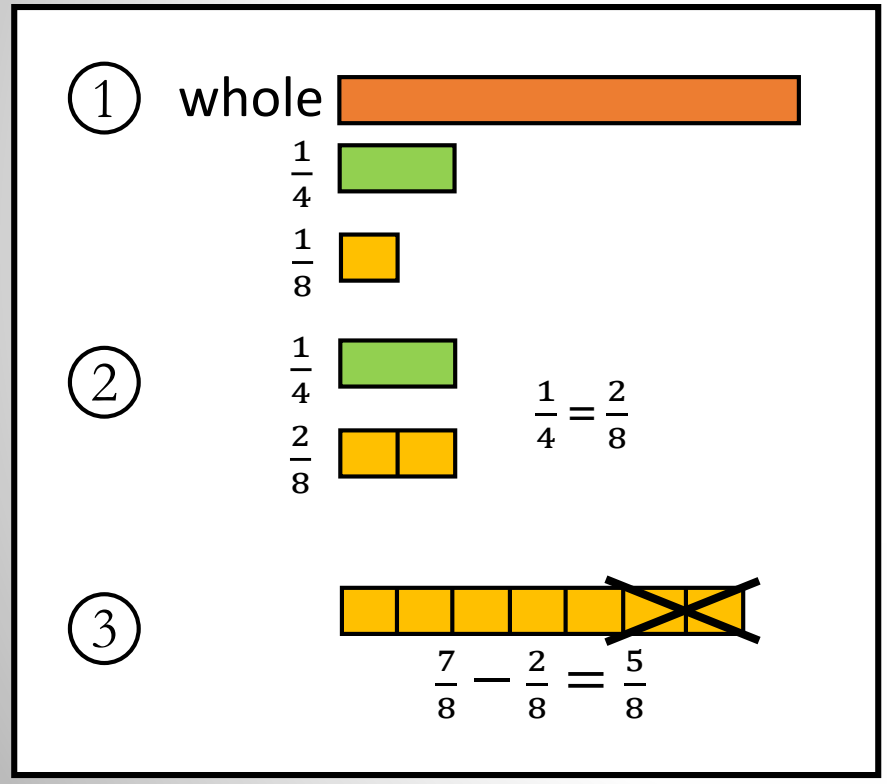
$\frac{1}{5}$ is close to 0 because 1 is relatively small compared to 5.

$\frac{10}{12}$ is close to 1 because 10 is relatively large compared to 12.

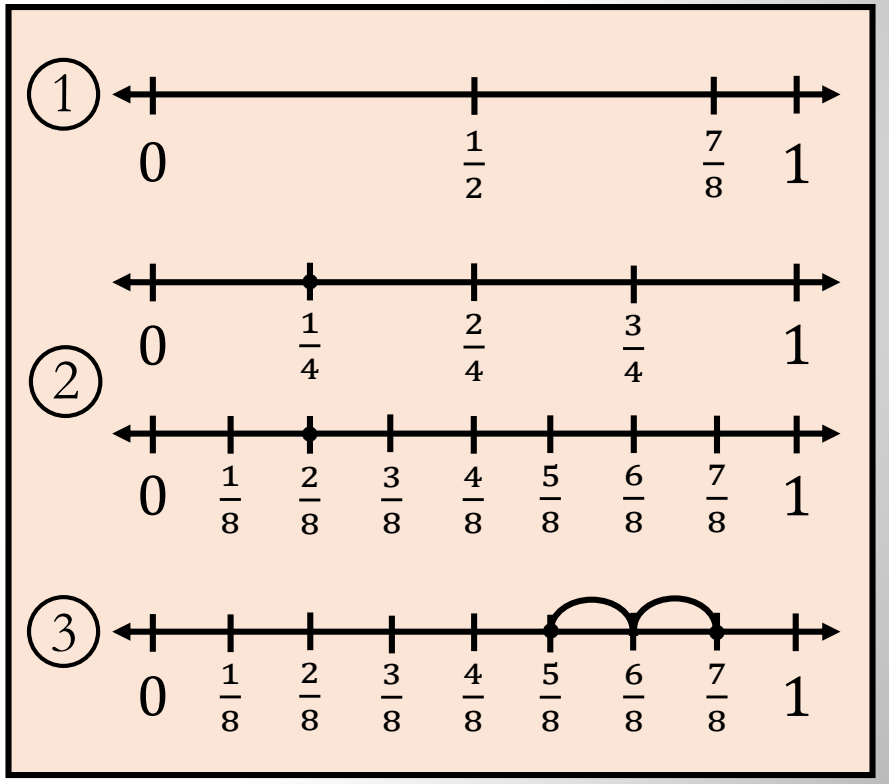
Therefore $\frac{1}{5} < \frac{10}{12}$.

SUBTRACTION PROBLEM

Cuisenaire Rods



Number Line



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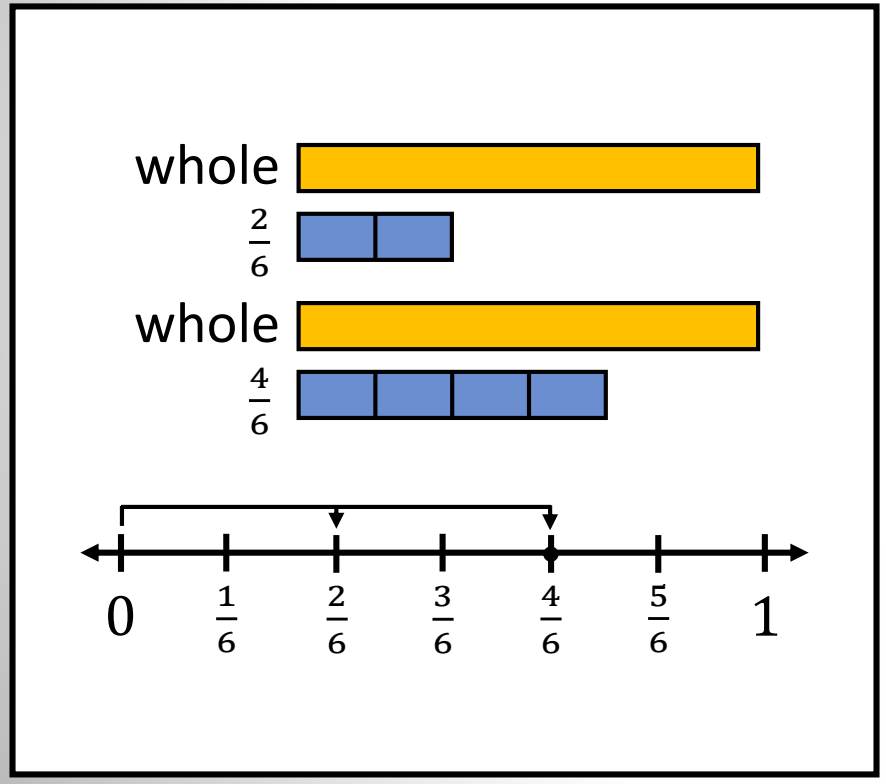
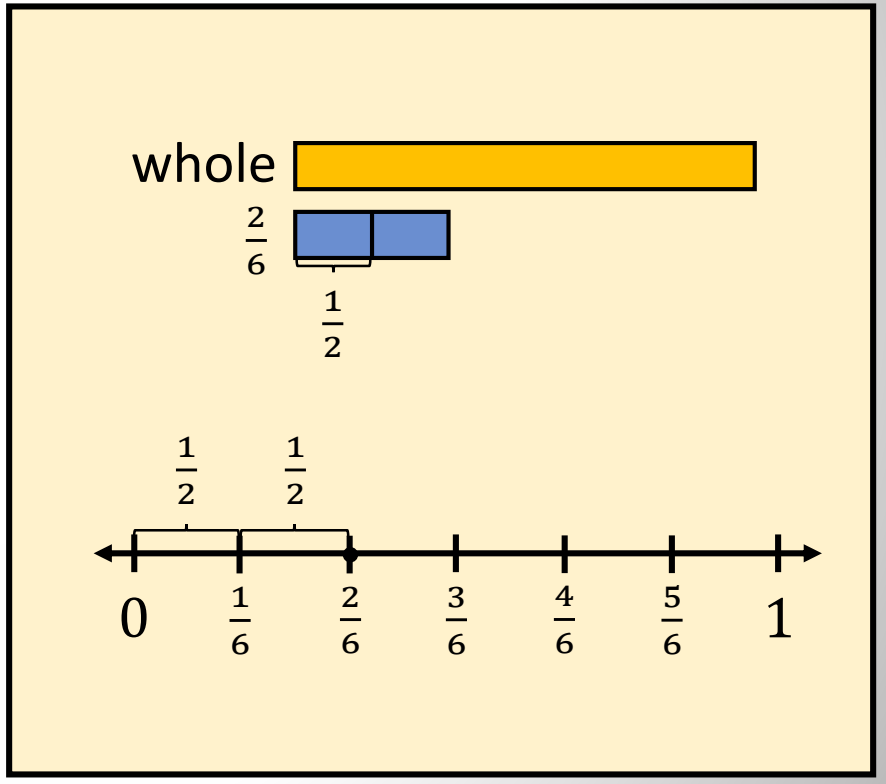
MULTIPLICATION PROBLEMS

$2 \times \frac{2}{6}$



$\frac{1}{2} \times \frac{2}{6}$



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OTHER ASPECTS OF THE FRACTIONS INTERVENTION

- Extensive practice
- Frequent and immediate feedback
- Opportunities for oral and written explanations
- Explanations leveraged through question shells by Ball and Shaughnessy

ABOUT THE RCT

- <u>205 5th graders</u>
  15th – 38th percentile on a validated fractions measure aligned with contemporary state standards
- <u>3 school districts from CA and TN</u>
  14 schools, 35 classrooms
- <u>Randomly assigned students to</u>
  Treatment = *TransMath* Fractions Intervention
  Control = what is currently offered by the school

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IMPLEMENTATION

Small-group intervention (n = 5)

Provided 3-4 times per week (52 lessons) for 35 minutes

Remedial and grade-level material

Interventionists (retired teachers, math tutors)

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IMPLEMENTATION

Review
(5 minutes)

Explicit Instruction
(10 minutes)

Guided Practice
(10 minutes)

Problem Solving
(10 minutes)

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WHAT DID WE FIND

- Measures of Fractions Knowledge

  TUF-4 (IRG, 2014): Hedges' *g* = .77

  TUF-5 (IRG, 2015): *g* = .65

- Measure of Fractions Procedures

  Fractions Procedures Test (Jordan et al., 2013): *g* = 1.04

- Measures of Magnitude Understanding

  NLE 0-1 (Siegler & Opfer, 2003): *g* = 1.25

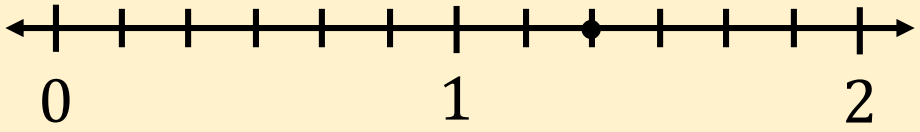
  NLE 0-2 (Siegler & Opfer, 2003): *g* = .84

*ALL significant at $p < .0001$

MEASURES OF FRACTION KNOWLEDGE
TUF-5 (IRG, 2015)

Hedges' g = .65, p < .0001

The point on the number line shows the value of the sum of two fractions.



Which expression has the same sum?

A. $\frac{4}{3}+\frac{4}{3}$

B. $\frac{6}{4}+\frac{2}{4}$

C. $\frac{5}{6}+\frac{3}{6}$

D. $\frac{2}{12}+\frac{6}{12}$

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MEASURE OF FRACTIONS PROCEDURES
FRACTIONS PROCEDURES TEST (JORDAN ET AL., 2013)

Hedges' g = 1.04, p < .0001

$3\frac{3}{8} + \frac{2}{8} =$

$1\frac{3}{4} - \frac{1}{4} =$

$\frac{5}{6} \times \frac{3}{4} =$

$\frac{1}{3} \div 4 =$

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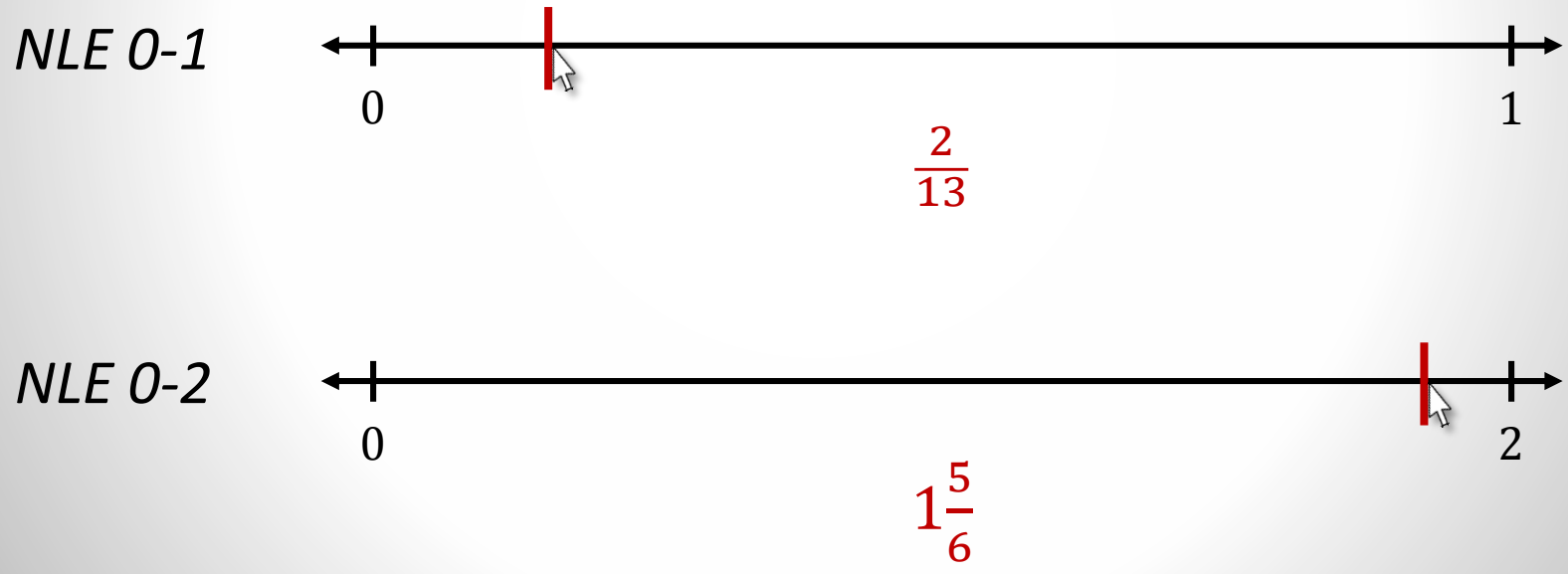
MEASURES OF MAGNITUDE UNDERSTANDING

NLE 0-1 (Siegler & Opfer, 2003): $g = 1.25$, $p < .0001$

NLE 0-2 (Siegler & Opfer, 2003): $g = .84$, $p < .0001$



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ASSESSMENT OF PERFORMANCE AND UNDERSTANDING

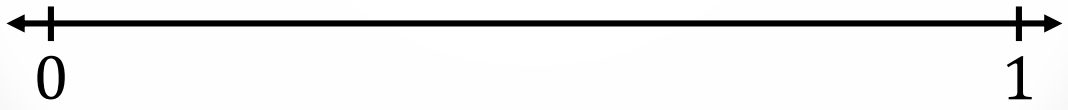
(aligned with contemporary state standards)

Order the fractions from least to greatest on the blank spaces below. Then, mark and label these fractions on the number line.

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{1}{8}$ $\frac{7}{12}$

_____ _____ _____ _____

least greatest



How did you know where to place each fraction? Explain your thinking.

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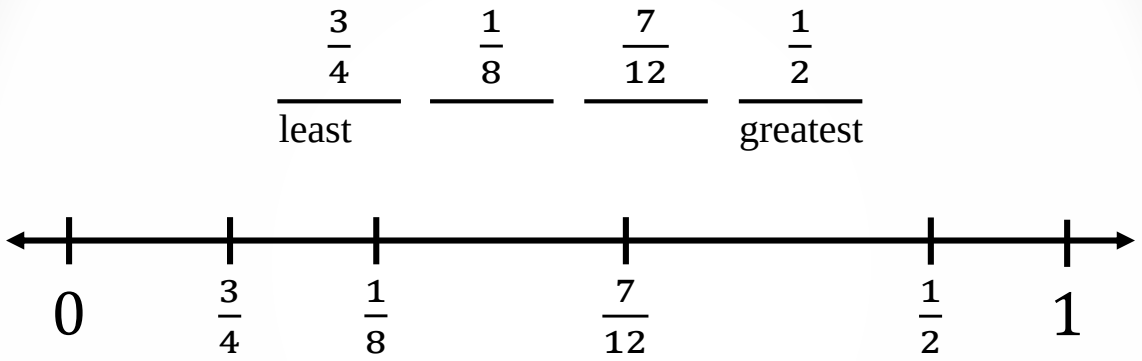
PERFORMANCE ASSESSMENT

Student A



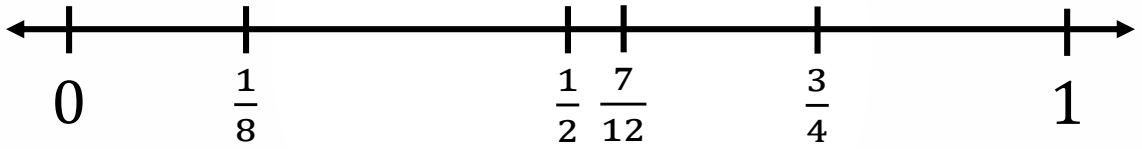
"I kinda help if you round you could see the smallest and the biggest."

PERFORMANCE ASSESSMENT

Student B

$\frac{1}{8}$ $\frac{1}{2}$ $\frac{7}{12}$ $\frac{3}{4}$

least greatest



"I used the number line and I looked at the <u>fractions</u> to see there <u>reltive size</u>. I put $\frac{1}{8}$ closer to 0 because it is a <u>unit fractions</u>. I put $\frac{1}{2}$ in the middle because it is a <u>benchmark</u> number and I put $\frac{7}{12}$ close to $\frac{6}{12}$ which is <u>equivalent</u> to $\frac{1}{2}$. I put $\frac{3}{4}$ closer to 1 because it is close to $\frac{4}{4}$ but not there yet. That is why I put the fractions there."

OUR RESEARCH TEAM

Russell Gersten

Karen Karp

Joseph Dimino

Keith Smolkowski

Kelly Haymond

Pam Foremski

Samantha Spallone

Christopher Tran

Instructional Research Group

RI MTSS

MULTI–TIERED SYSTEM of SUPPORT

Math Intervention in Rhode Island's Secondary Schools

Nicole Bucka, District RTI Coordinator
Cumberland School District, Rhode Island
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Twitter @nbucka

Paul V. Sherlock Center on Disabilities

RIDE Rhode Island Department of Education

Goal: Implement IES Practice Guide for Math RTI

Recommendation	Level of evidence
Tier 1	
1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	Moderate
Tiers 2 and 3	
2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	Low
3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	Strong
4. Interventions should include instruction on solving word problems that is based on common underlying structures.	Strong
5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	Moderate
6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	Moderate
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Low
8. Include motivational strategies in tier 2 and tier 3 interventions.	Low

Source: Authors' compilation based on analysis described in text.

Lesson Learned #1

Intervention Content/Focus:

Rational Numbers
Versus
Whole Numbers

Lesson Learned #2

Intervention Pedagogy:

CRA 

Students **without** disabilities

Explicit, Systematic Direct Instruction

Students **with** disabilities

Training Sequence

Intervention Training 1: Overview Continuum, Struggling Learners, Behavior and Motivation

Intervention Training 2: Entrance/Exit Decision Rules, Standard Protocol Tier 2 Intervention

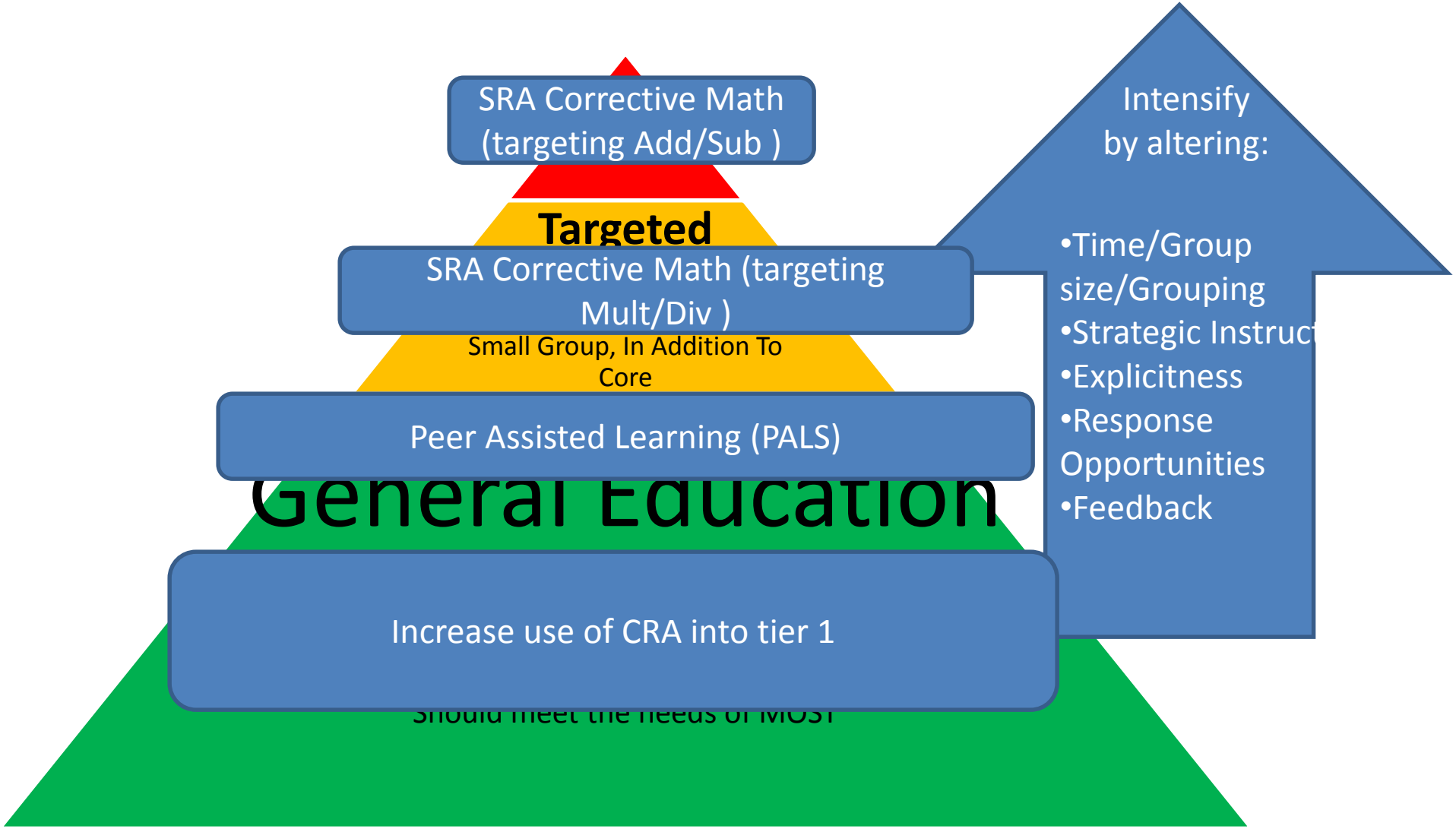
Training for all math teachers: C-R-A integers, place value, multiplication

Intervention Training 3: Data Based Individualization using Progress Monitoring & Evaluating Effectiveness

Training for all math teachers: C-R-A in divisibility, factors, rational #s

https://www.sophia.org/tutorials/ri-mtss-math-intervention-training-series

Full Continuum



Lesson Learned #3

Intervention Pedagogy:

Behavior, Motivation, and Self-Regulation are key to struggling learners in mathematics

Training Sequence

Intervention Training 1: Overview Continuum, Struggling Learners, Behavior and Motivation

Intervention Training 2: Entrance/Exit Decision Rules, Standard Protocol Tier 2 Intervention

Training for all math teachers: C-R-A integers, place value, multiplication

Intervention Training 3: Data Based Decision Making using Progress Monitoring & Evaluating Effectiveness

Training for all math teachers: C-R-A in divisibility, factors, rational #s

https://www.sophia.org/tutorials/ri-mtss-math-intervention-training-series

> Current Issue > vol. 107 no. 5 > Sian L. Beilock, 1860–1863, doi: 10.1073/pnas.0910967107

CrossMark click for updates

Female teachers' math anxiety affects girls' math achievement

Sian L. Beilock [1], Elizabeth A. Gunderson, Gerardo Ramirez, and Susan C. Levine

Teach Positive "Self Talk"

Title: The Role of Instructors in Creating Math Anxiety in Students from Kindergarten through College
Author(s): Carol D. Jackson and R. Jon Leffingwell
Source: The Mathematics Teacher, Vol. 92, No. 7 (October 1999), pp. 583-586
Stable URL: http://www.jstor.org/stable/27971118

Research shows that the fourth grade is often when students first experience math anxiety (Tankersley

Web address:
http://www.sciencedaily.com/releases/2012/10/121031213711.htm

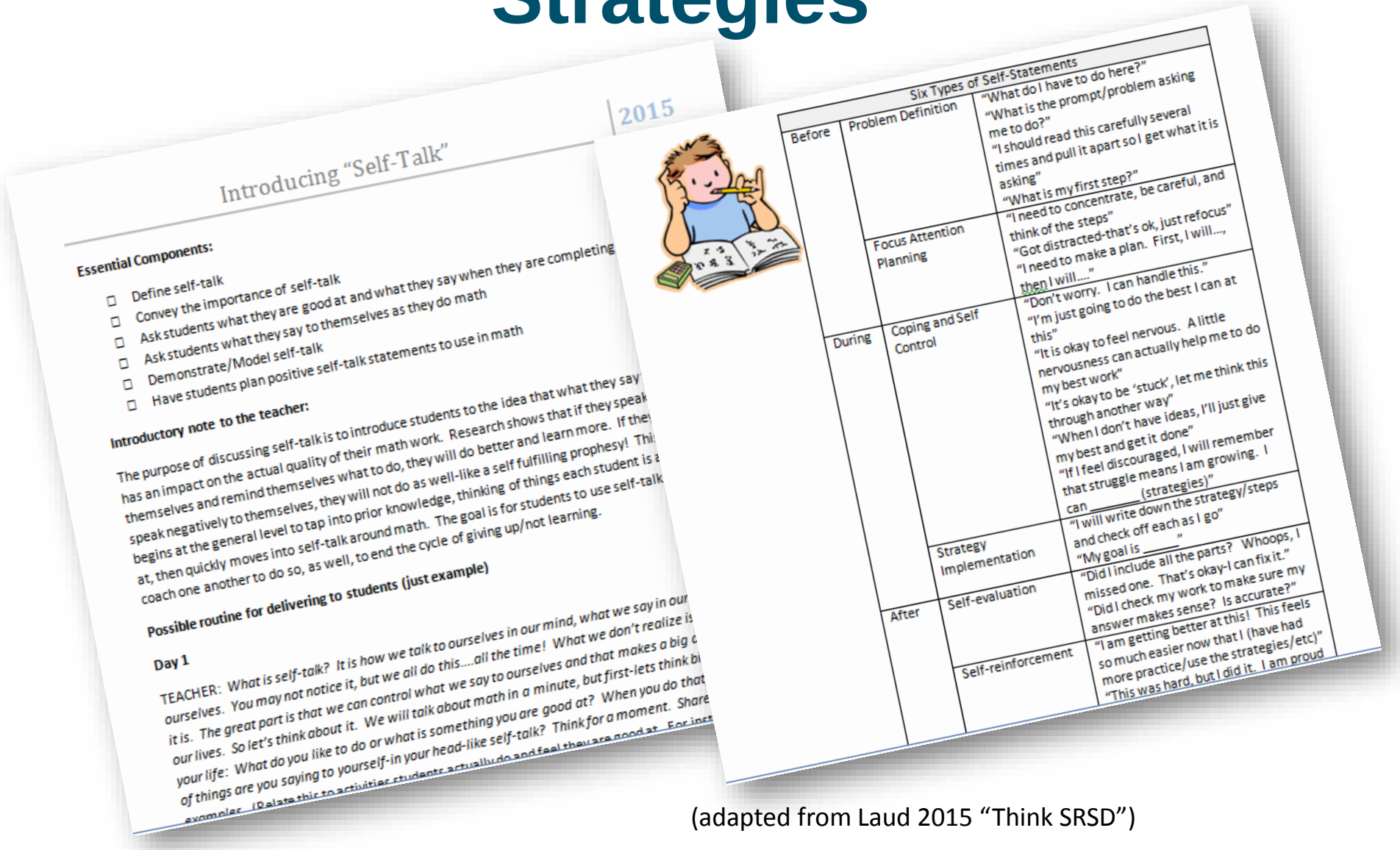
When People Worry About Math, the Brain Feels the Pain

UChicago researchers have found that the higher a person's anxiety about math, the more anticipating math activated areas of the brain related to experiencing pain.

Oct. 31, 2012 — Mathematics anxiety can prompt a response in the brain similar to when a person experiences physical pain, according to new research at the University of Chicago.

RI MTSS

Teaching Tools: Lesson & Self Talk Strategies



Introducing "Self-Talk"

2015

Essential Components:

- ☐ Define self-talk
- ☐ Convey the importance of self-talk
- ☐ Ask students what they are good at and what they say when they are completing
- ☐ Ask students what they say to themselves as they do math
- ☐ Demonstrate/Model self-talk
- ☐ Have students plan positive self-talk statements to use in math

Introductory note to the teacher:

The purpose of discussing self-talk is to introduce students to the idea that what they say has an impact on the actual quality of their math work. Research shows that if they speak themselves and remind themselves what to do, they will do better and learn more. If they speak negatively to themselves, they will not do as well-like a self fulfilling prophesy! This begins at the general level to tap into prior knowledge, thinking of things each student is at, then quickly moves into self-talk around math. The goal is for students to use self-talk coach one another to do so, as well, to end the cycle of giving up/not learning.

Possible routine for delivering to students (just example)

Day 1

TEACHER: *What is self-talk? It is how we talk to ourselves in our mind, what we say in our* ourselves. You may not notice it, but we all do this....all the time! What we don't realize is it is. The great part is that we can control what we say to ourselves and that makes a big our lives. So let's think about it. We will talk about math in a minute, but first-lets think b your life: What do you like to do or what is something you are good at? When you do that of things are you saying to yourself-in your head-like self-talk? Think for a moment. Share examples. (Relate this to activities students actually do and feel they are good at. For inst

Six Types of Self-Statements		
Before	Problem Definition	"What do I have to do here?" "What is the prompt/problem asking me to do?" "I should read this carefully several times and pull it apart so I get what it is asking"
	Focus Attention Planning	"What is my first step?" "I need to concentrate, be careful, and think of the steps" "Got distracted-that's ok, just refocus" "I need to make a plan. First, I will...., then I will...."
During	Coping and Self Control	"Don't worry. I can handle this." "I'm just going to do the best I can at this" "It is okay to feel nervous. A little nervousness can actually help me to do my best work" "It's okay to be 'stuck', let me think this through another way" "When I don't have ideas, I'll just give my best and get it done" "If I feel discouraged, I will remember that struggle means I am growing. I can _____(strategies)"
	Strategy Implementation	"I will write down the strategy/steps and check off each as I go" "My goal is _____"
After	Self-evaluation	"Did I include all the parts? Whoops, I missed one. That's okay-I can fix it." "Did I check my work to make sure my answer makes sense? Is accurate?"
	Self-reinforcement	"I am getting better at this! This feels so much easier now that I (have had more practice/use the strategies/etc)" "This was hard, but I did it. I am proud

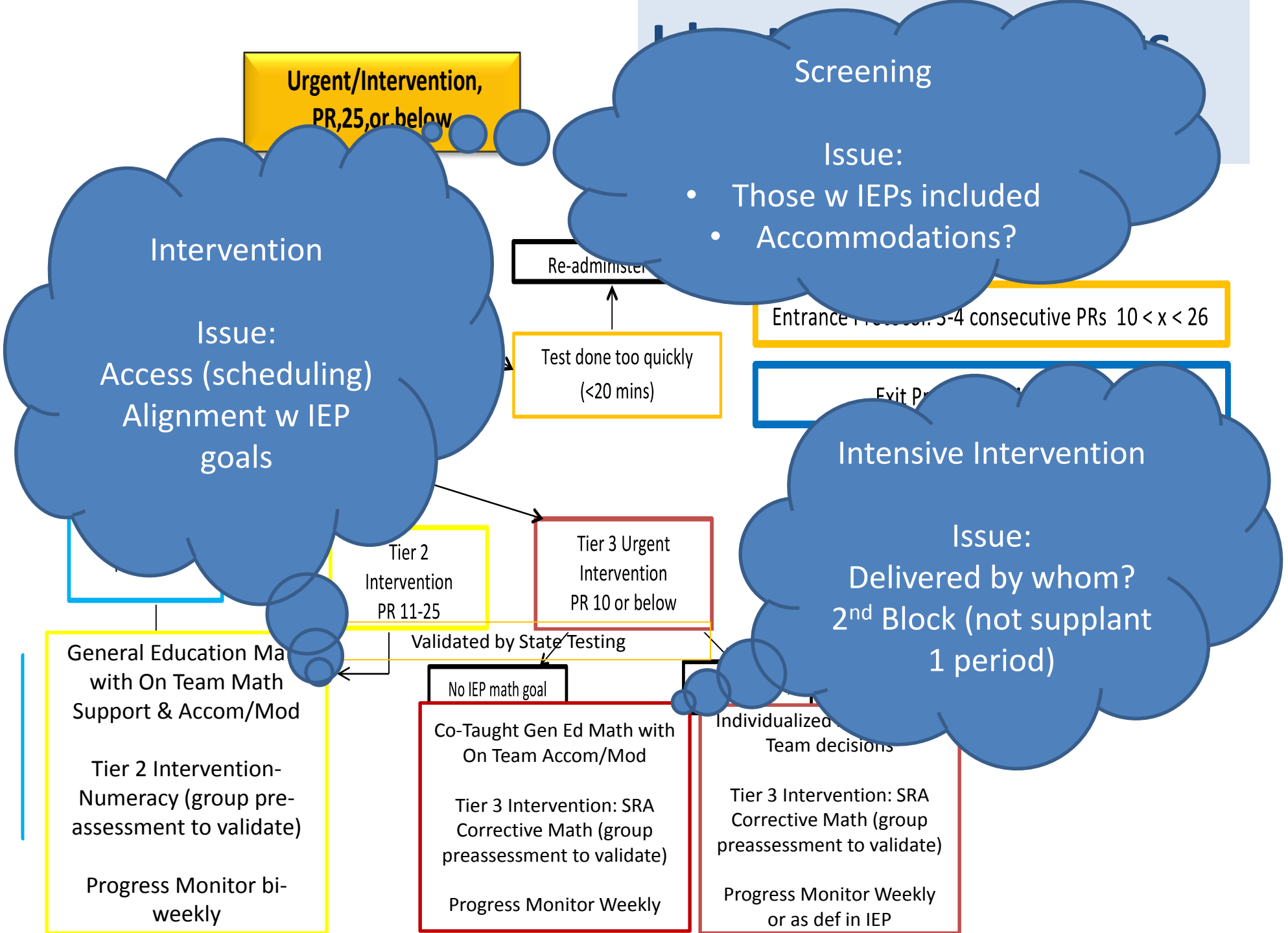
(adapted from Laud 2015 "Think SRSD")

RI MTSS

Lesson Learned #4

Systems:

Intervention and Special Education - A Hot Mess


Urgent/Intervention, PR,25,or below
Screening
Issue:
• Those w IEPs included
• Accommodations?
Intervention
Issue:
Access (scheduling)
Alignment w IEP goals
Re-administer
Test done too quickly (<20 mins)
Entrance Protocol: 3-4 consecutive PRs 10 < x < 26
Exit Pr
Intensive Intervention
Issue:
Delivered by whom?
2nd Block (not supplant 1 period)
Tier 2 Intervention PR 11-25
Tier 3 Urgent Intervention PR 10 or below
Validated by State Testing
General Education Ma with On Team Math Support & Accom/Mod
Tier 2 Intervention- Numeracy (group pre-assessment to validate)
Progress Monitor bi-weekly
No IEP math goal
Co-Taught Gen Ed Math with On Team Accom/Mod
Tier 3 Intervention: SRA Corrective Math (group preassessment to validate)
Progress Monitor Weekly
Individualized
Team decisions
Tier 3 Intervention: SRA Corrective Math (group preassessment to validate)
Progress Monitor Weekly or as def in IEP

Whose role is it, anyway?

Roles Defined

Interventionist	Special Educator


Results

Tier 3 Mathematics Intervention in Elementary School: What Should it Look Like?

Rebecca Zumeta Edmonds and Kathleen Hughes Pfannenstiel

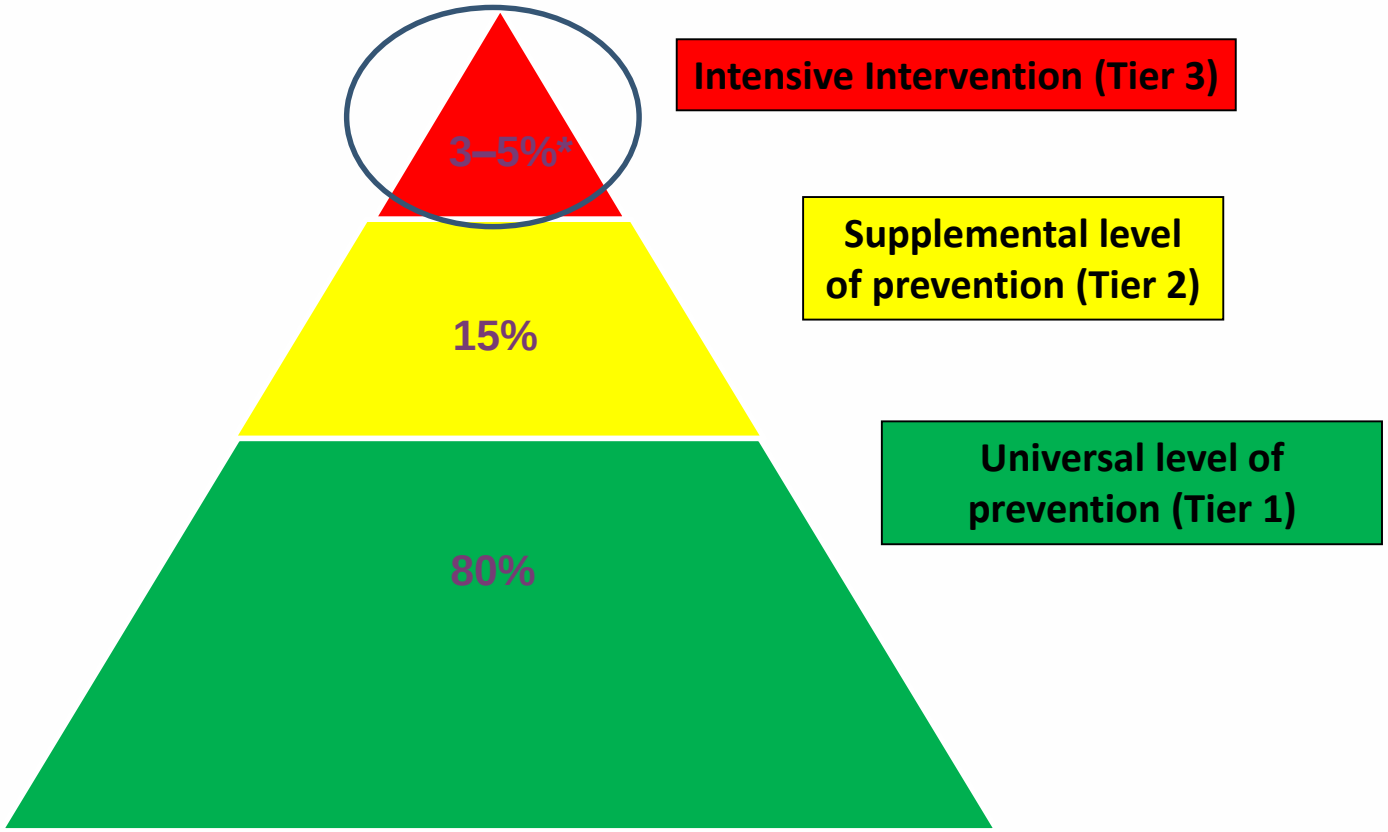
American Institutes for Research

Today's Presentation

- Brief Project Overview
- Key Elements of Data-based Individualization
- Lessons from Implementation
- Questions & Discussion

FRANKLIN PIERCE SCHOOLS

Multi-Tiered System of Support (MTSS)



Information from www.rti4success.org

*Including students with and at risk for disabilities

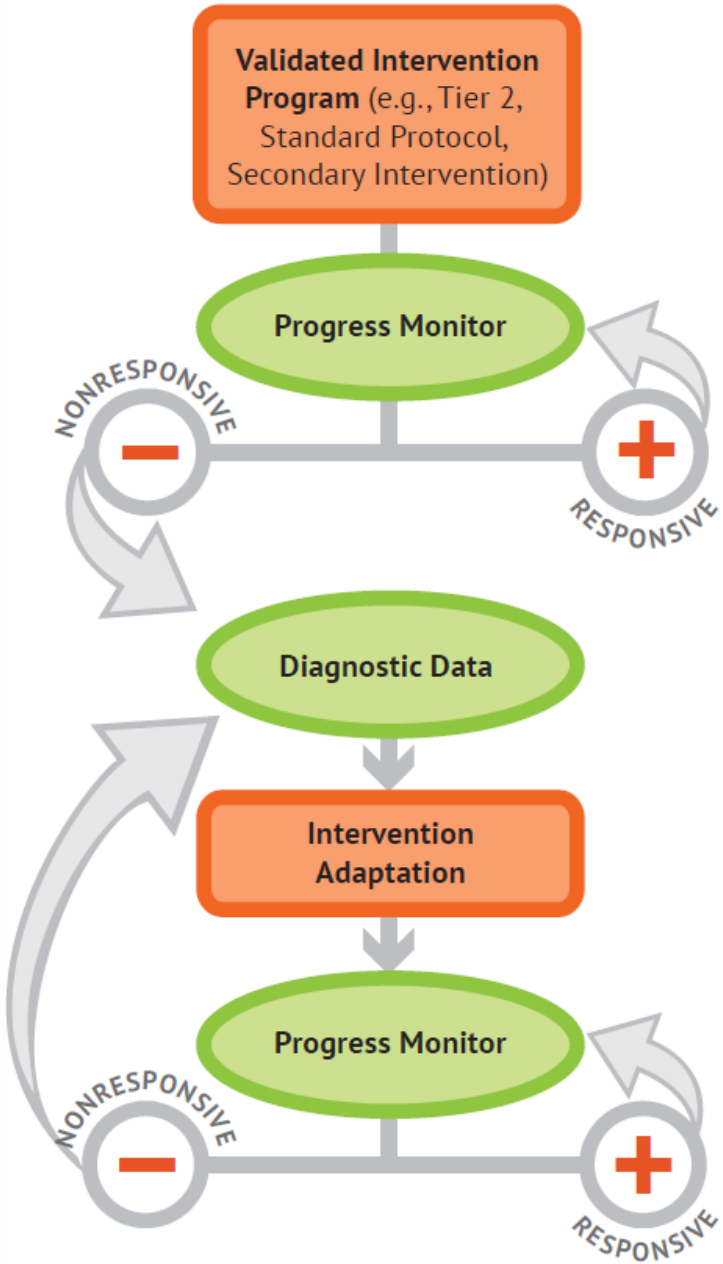
What is intensive intervention?

Intensive intervention is designed to address *severe and persistent* learning or behavior difficulties. Intensive interventions should be:

(a) Driven by data

(b) Characterized by increased intensity (e.g., smaller group, expanded time) and individualization of academic instruction and/or behavioral supports

FRANKLIN PIERCE SCHOOLS



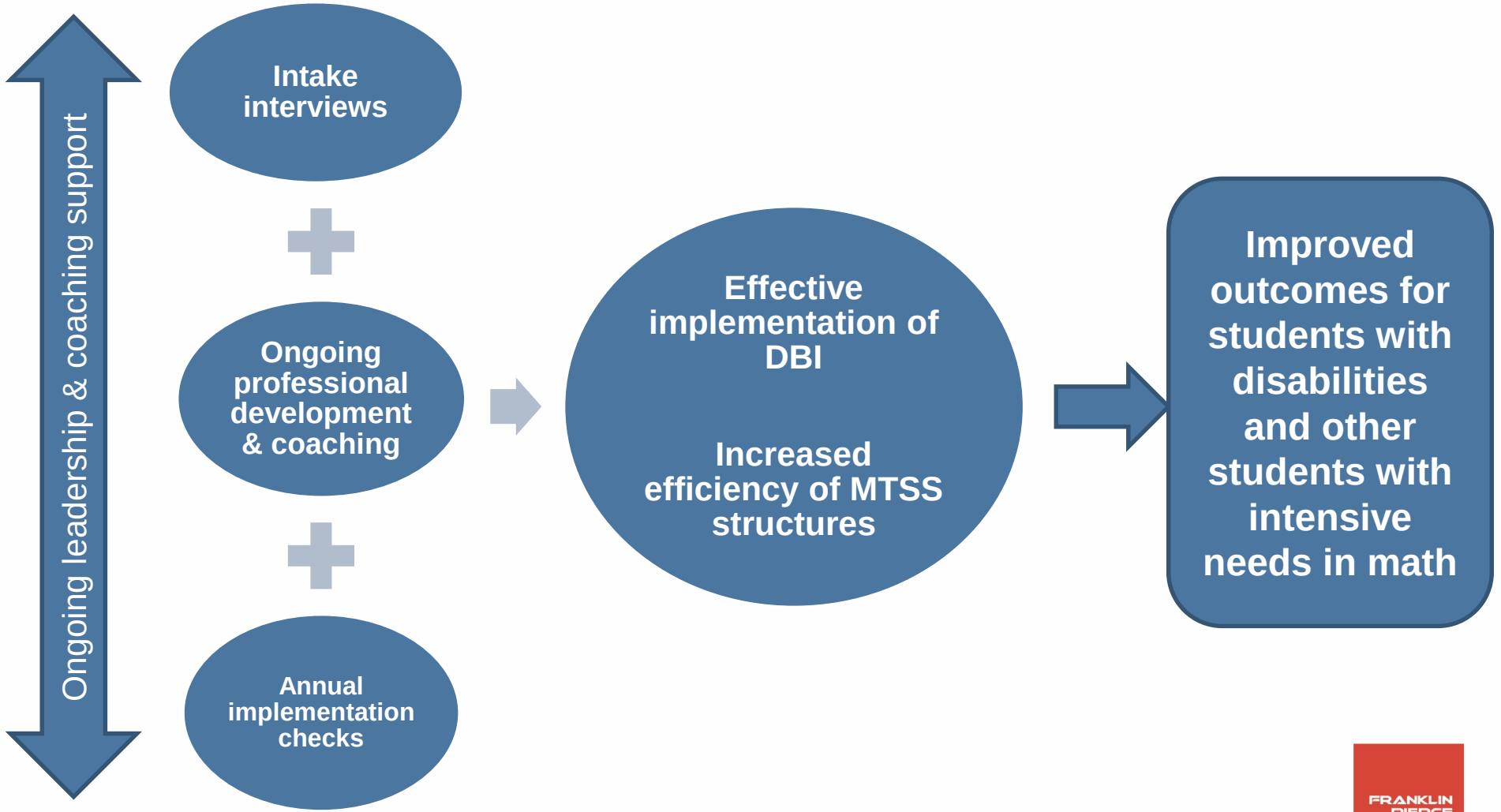
The DBI Process

1. Secondary intervention program, delivered with greater intensity
2. Progress monitoring
3. Diagnostic data
4. Adaptation
5. Continued progress monitoring, with adaptations as needed to ensure adequate progress

FRANKLIN PIERCE SCHOOLS

Putting it Together: Implementation in Real Life

Implementation



Content of Mathematics PD

Assessment for DBI	Intervention Design in Mathematics
• Progress monitoring in mathematics • Informal diagnostic assessment in mathematics • Using data for different purposes	• Standardized Tier 2 protocols in mathematics • Adapting interventions • Standards-aligned modules addressing common areas students struggle (e.g., place value, fractions, word problems)

For examples, visit: http://www.intensiveintervention.org/content/dbi-training-series; http://www.intensiveintervention.org/standards-relevant-instruction-multi-tiered-systems-support-mtss-or-response-intervention

Implementation Supports Emphasize the System Surrounding the Student

Other tiers of MTSS and role of special education

Staff math intervention knowledge and skills

School and district support

Use of assessments for different purposes

Schedule provides time for math intervention, data review, and planning

Behaviors impact academic performance, and vice versa

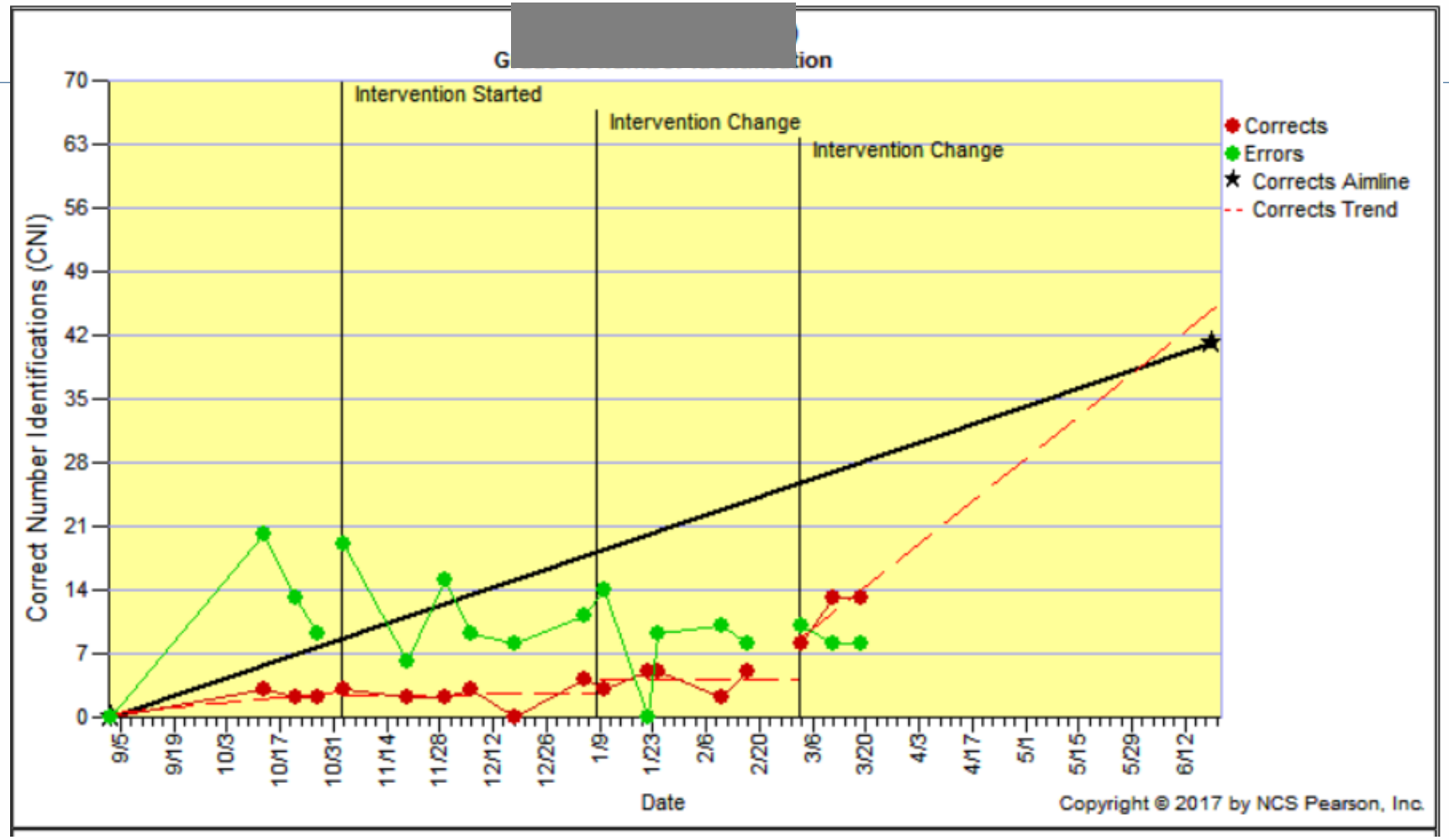
Consistent messaging about expectations

Fidelity, follow through, and re-teaching

Student Data: Kindergartener

- Tier 3 after Fall benchmarking
  - Goal: Grade level ROI
- Intervention 1: Flash Cards 25 minutes a week with para
  - Little to no growth
- Intervention 2: Explicit instruction in a small group with one to one correspondence and number ID to 5
  - 40 minutes a week (every other day for 20 min)
  - Little to no growth
- Intervention 3: More time (+ 40 minutes a week)
  - Increased time to 20 min daily
  - Still not making enough growth and only identifying numbers 1-3
- Intervention 4: Explicit Direct Instruction program
  - Progress evident, can identify numbers 1-5

Grade K: Number Identification



Words to the Wise

- You will have to repeat yourself… a lot.
- You will have to repeat yourself… a lot.
- Explicitly address behavior and how it may manifest in academic domains.
- Implementation of MTSS may not need to be linear.
- Fidelity matters: People don't always do what you tell them to do…even if you really want them to.

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