Teaching Fraction Concepts to At-Risk Fifth Graders Using the Number Line and Cuisenaire Rods

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Instructional Research Group

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Fraction Intervention

- Goal: Build fifth grade struggling students' conceptual and procedural knowledge of foundational as well as grade level fractions content.
- Curriculum: 54 35-minute fractions lessons from *TransMath*® (Level 2; Woodward & Stroh, 2016).
- Fractions Content: (a) Aligned with fourth and fifth grade Commons Core State Standards in mathematics; (b) focus on developing student understanding in addition to procedural competence; (c) addressed foundational fraction concepts as well concepts underlying the four operations.
- Key Representations: Number lines and Cuisenaire rods.

Pilot Study

- 15 at-risk students were randomly assigned to treatment (fractions Intervention) and control (business as usual; 10 treatment, 5 control).
- Intervention was provided 4 days week.
- Each intervention group included 5 students.
- Tutors were research staff with experience in teaching struggling students.

Results

- Overall, students who received the fractions intervention (n=10) did better than students who did not receive the fractions intervention (n=5). See table below.
- With a small sample size, these should be interpreted with caution.

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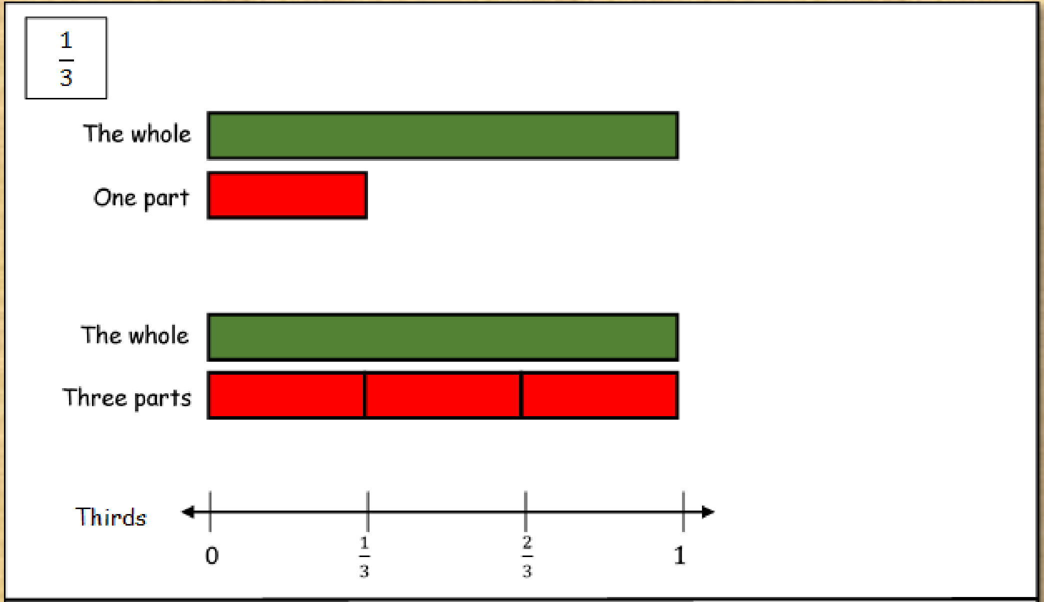
Student Performance on Fractions Post-test Measures

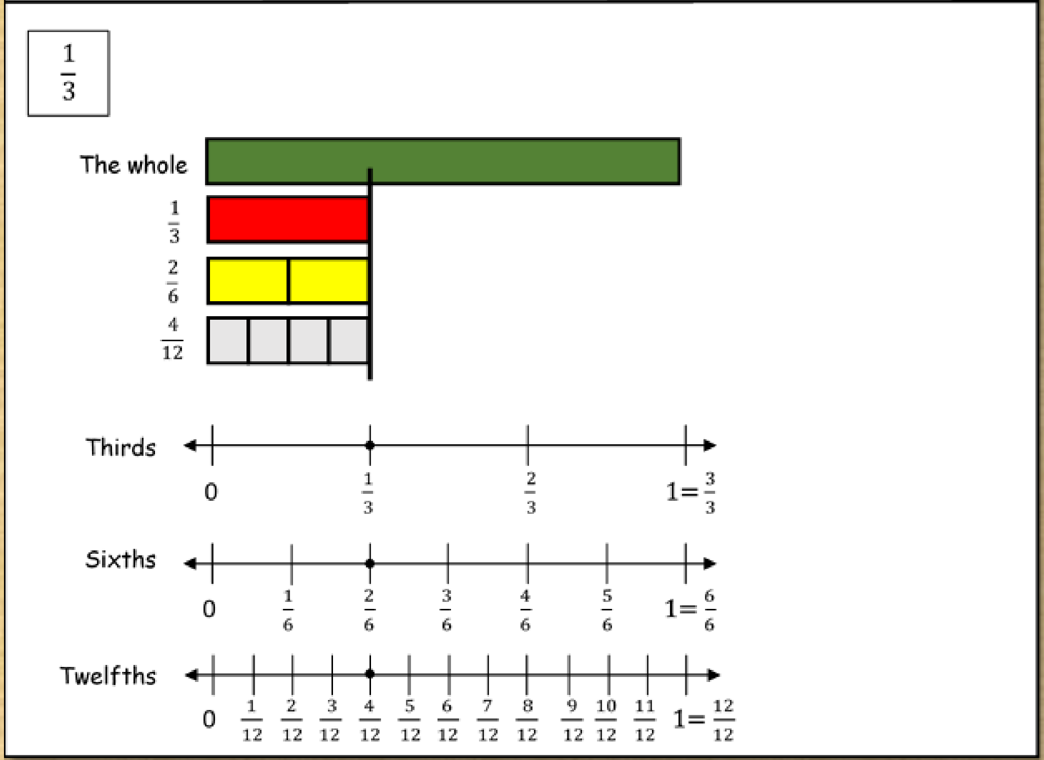
Measure	Treatment Post-test (*n* = 9)		Control Post-test (*n* = 5)		Hedges' *g*
	Mean	*SD*	Mean	*SD*	
Test of Understanding Fractions, Fourth-Grade[a]	18.22	2.95	12.60	5.46	1.418*
Test of Understanding Fractions, Fifth-Grade	11.89	2.42	8.80	3.90	1.031~
Curriculum Aligned Fraction Measure	19.56	2.24	12.20	5.85	1.915~
Test of Fractions Procedures	16.89	4.62	10.60	6.03	1.225*
Number Line Estimation 0-1[b]	3.97	0.77	20.21	8.67	3.277**
Number Line Estimation 0-2[c]	8.82	4.20	15.70	5.73	1.446*

~ Significant at p = .10; * significant at p = .05; ** significant at p = .01.

[a]Pretest mean for treatment group = 10.67 (SD = 1.32); pretest mean for control group = 10.40 (SD = 1.14). [b]For this measure, scores are a representation of percent absolute error; therefore, a low score corresponds to high performance. [c]For this measure, scores are a representation of percent absolute error; therefore, a low score corresponds to high performance.

Understanding Fractions & Equivalences



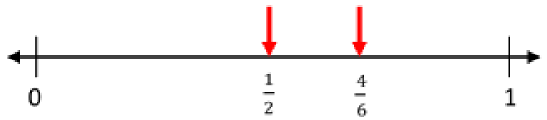


Number Line Estimation

Using Relative Size

Comparing Fraction to Benchmark

"I know $\frac{4}{6}$ is here because it is $\frac{1}{6}$ greater than $\frac{3}{6}$, which is equivalent to $\frac{1}{2}$."



Comparing Fractions Using Relative Size

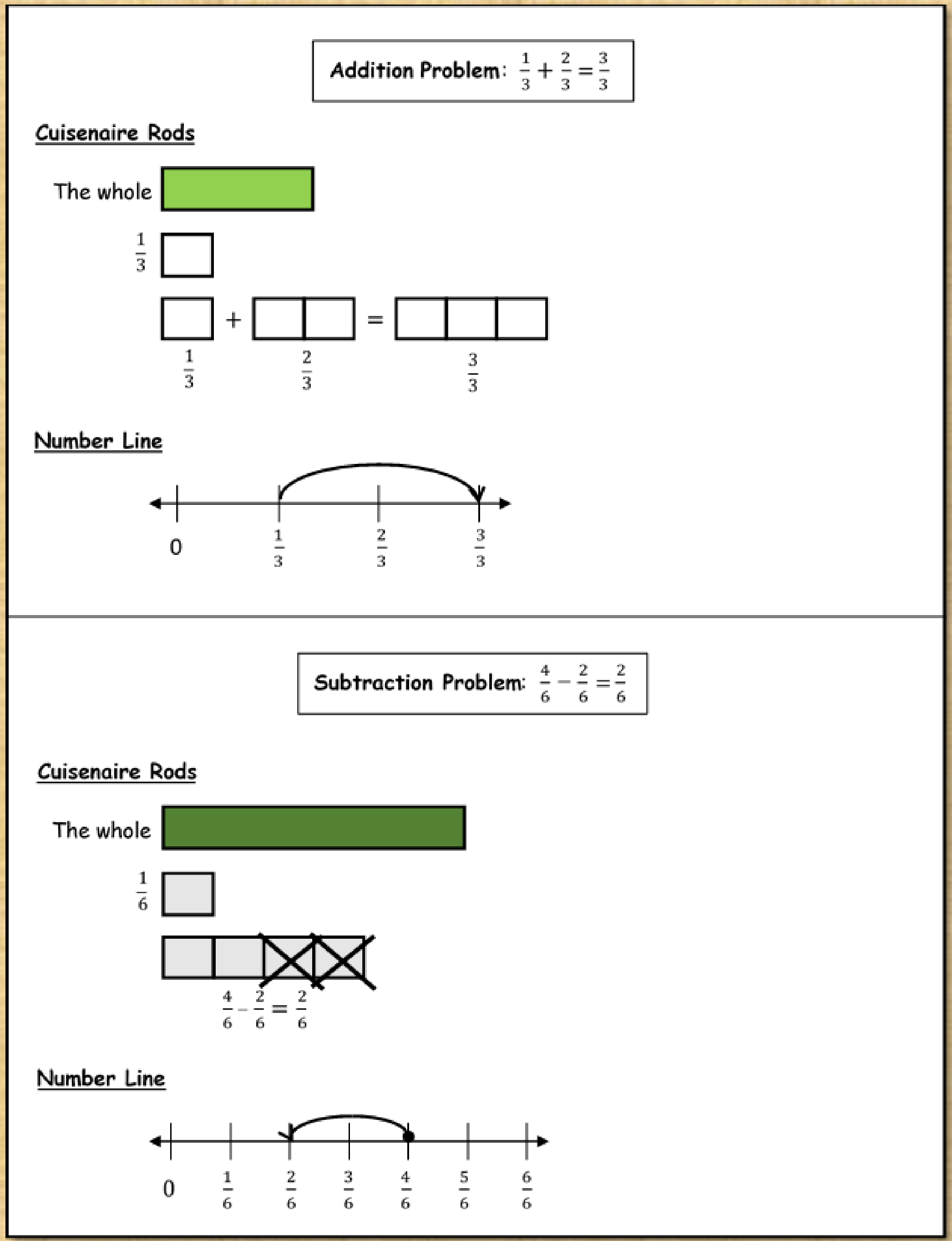
$$\frac{1}{5} < \frac{10}{12}$$

"$\frac{1}{5}$ is close to 0 because 1 is relatively small compared to 5.

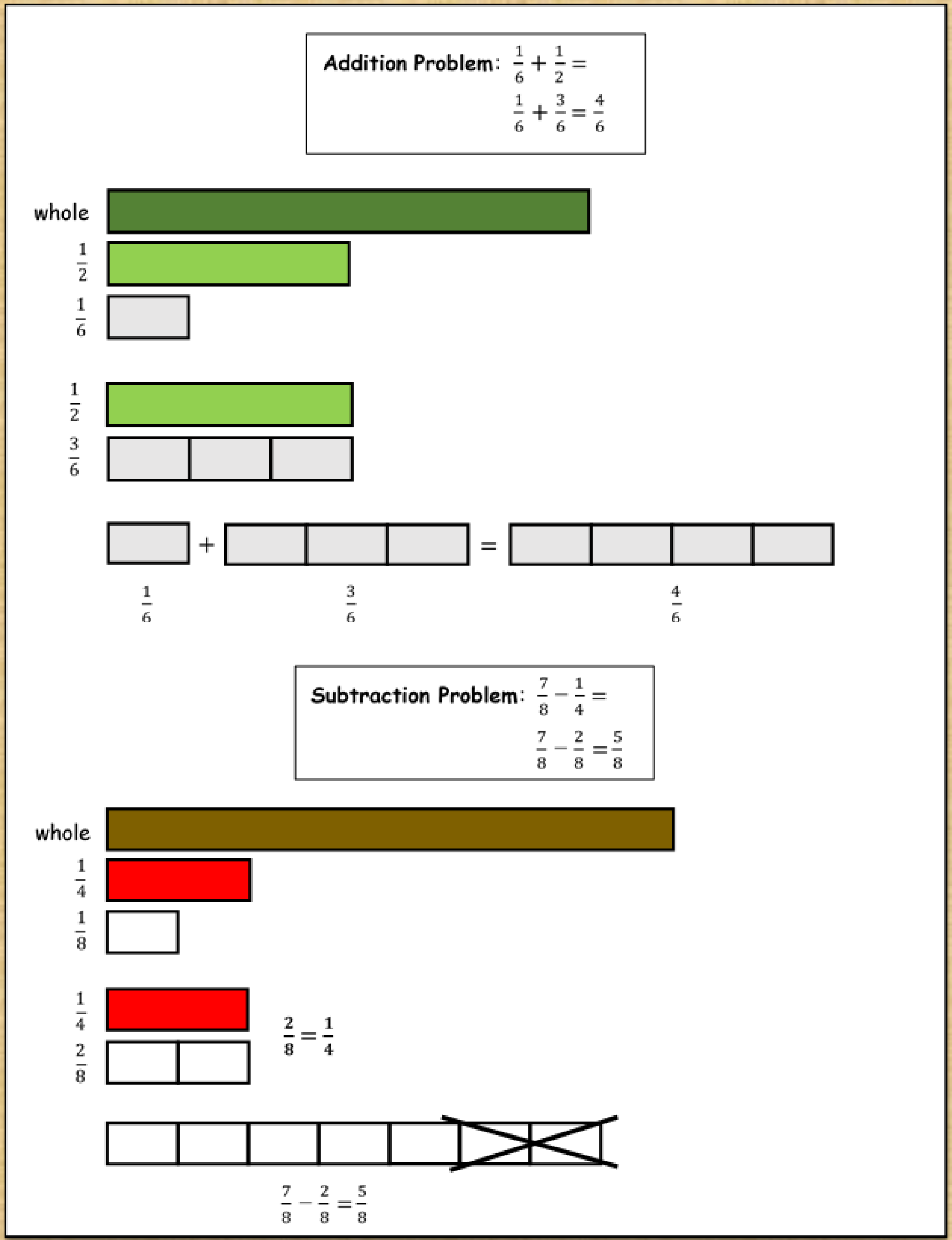
$\frac{10}{12}$ is close to 1 because 10 is relatively large compared to 12.

Therefore $\frac{1}{5} < \frac{10}{12}$."

Adding & Subtracting Fractions with Like Denominators



Adding & Subtracting fractions with Unlike Denominators

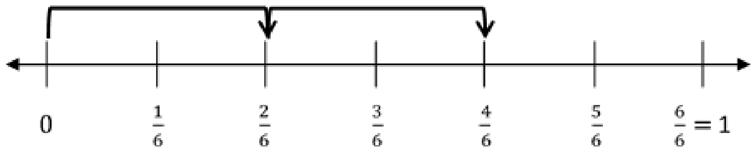


Understanding Multiplication Problems

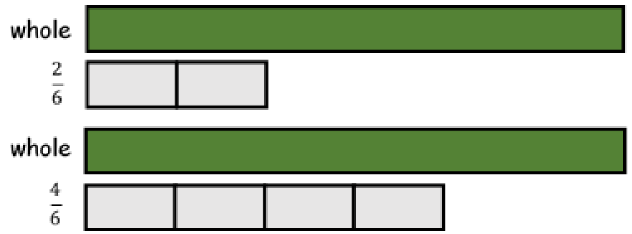
"Whole number times a fraction"

$2 \times \frac{2}{6}$ *or* 2 "groups of" $\frac{2}{6}$

Number Line



Cuisenaire Rods



Standard Algorithm

Multiple the numerators across:

Multiple the denominators across:

$$2 \times \frac{2}{6} = \frac{2}{1} \times \frac{2}{6} = \frac{4}{6}$$

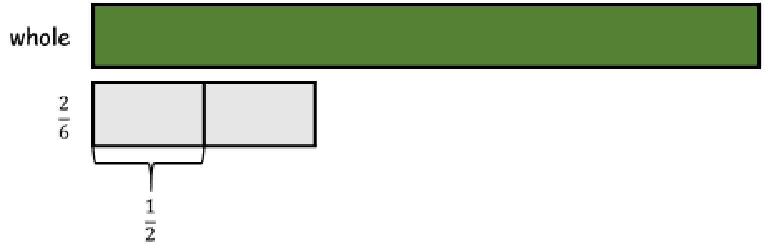
Then simplify: $\frac{4}{6} = \frac{2}{3}$

Scaffold Learning for Fraction Multiplication Problems

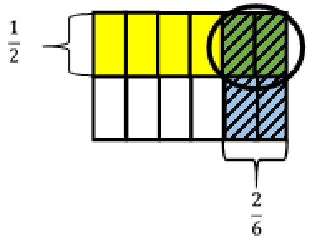
"Fraction times a fraction"

$\frac{1}{2} \times \frac{2}{6}$ or $\frac{1}{2}$ "of" $\frac{2}{6}$

Cuisenaire Rods



Area Model



Product of $\frac{1}{2} \times \frac{2}{6} = \frac{2}{12}$

Standard Algorithm

Multiply the numerators across:

Multiply the denominators across:

$\frac{1}{2} \times \frac{2}{6} = \frac{2}{12}$

Then simplify: $\frac{2}{12} = \frac{1}{6}$

Demonstration of the Pattern of Multiplication

The problem $\frac{1}{2} \times \frac{2}{6}$ is the same as $\frac{1}{2}$ of $\frac{2}{6}$.

Because $\frac{1}{2}$ of $\frac{2}{6}$ is less than $\frac{2}{6}$, the product will be less than $\frac{2}{6}$.

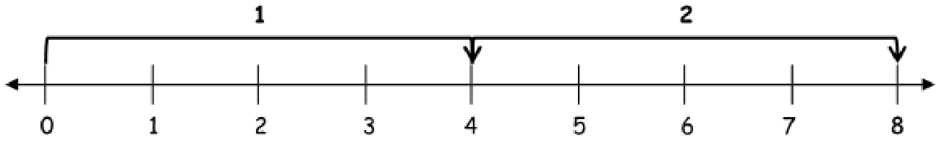
$\frac{1}{2} \times \frac{2}{6} = \frac{2}{12} = \frac{1}{6}$ — The product of $\frac{1}{6}$ is less than $\frac{2}{6}$.

Understanding Division Problems

Problem: 8÷2 = 4

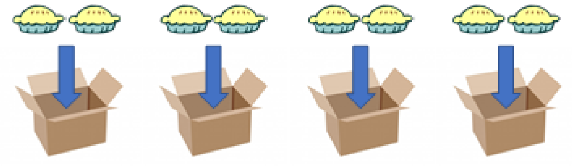
A. From the perspective of how many in a group:

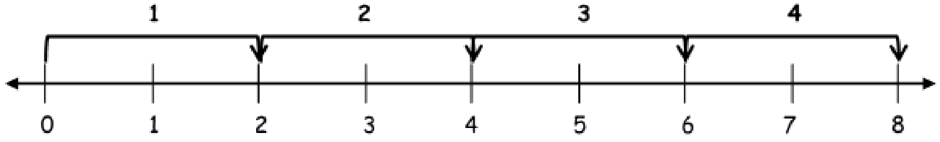
There are 8 pies. The pies are equally divided into two boxes. How many pies are in each box?



B. From the perspective of how many groups:

There are 8 pies. The pies are equally divided into boxes of two pies each. How many boxes are needed? (i.e., how many groups of 2 are there)?





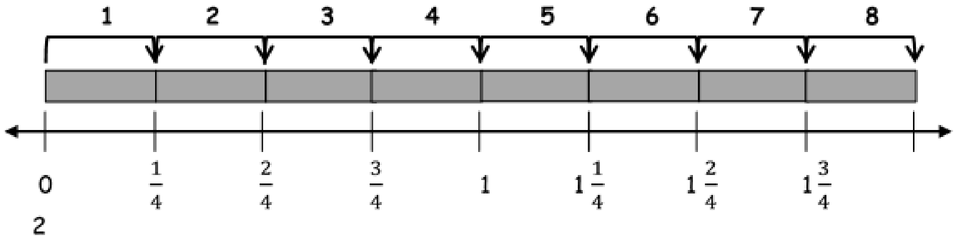
Scaffold Learning of Fraction Division Problems

Problem:

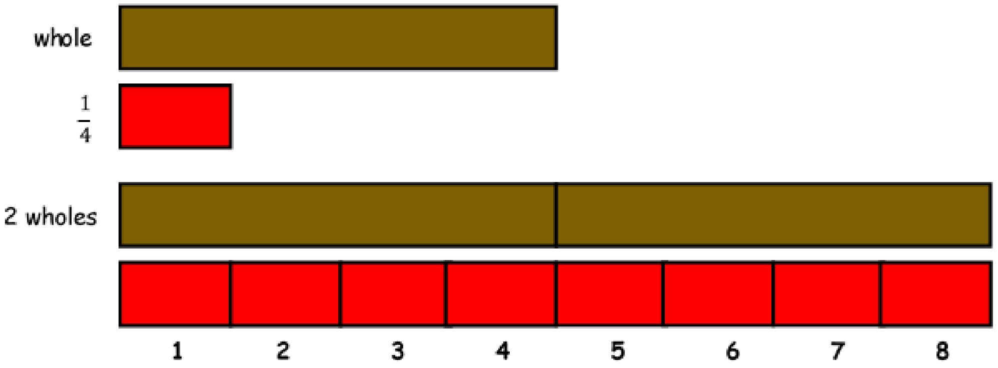
I have 2 pies. If I cut a pie into fourths, how many pieces of pie are there?

That is, how many groups of $\frac{1}{4}$ are there in 2? Or how many fourths are in two?

Number Line



Cuisenaire Rods



Standard Algorithm

$$2 \div \frac{1}{4} = \frac{2}{1} \times \frac{4}{1} = \frac{8}{1} = 8$$

Answer: There are 8 groups of $\frac{1}{4}$ in 2.