

Fraction Tutoring Training Day - 2

LESSONS FROM TRANS MATH LEVEL 2

Pretesting reminders/questions

1. Pretesting begins the week of Oct. 3
2. There are computer tests (individual) and 2 small-group pretests
3. You will pull kids for 5-10 minutes for assent and computer
4. You will pull kids in small-groups for the pretesting
5. You will make up absent students
6. We will assign one make-up tester per school (this will be on your schedule)

Handout Packet

iPhone Recorder Guide

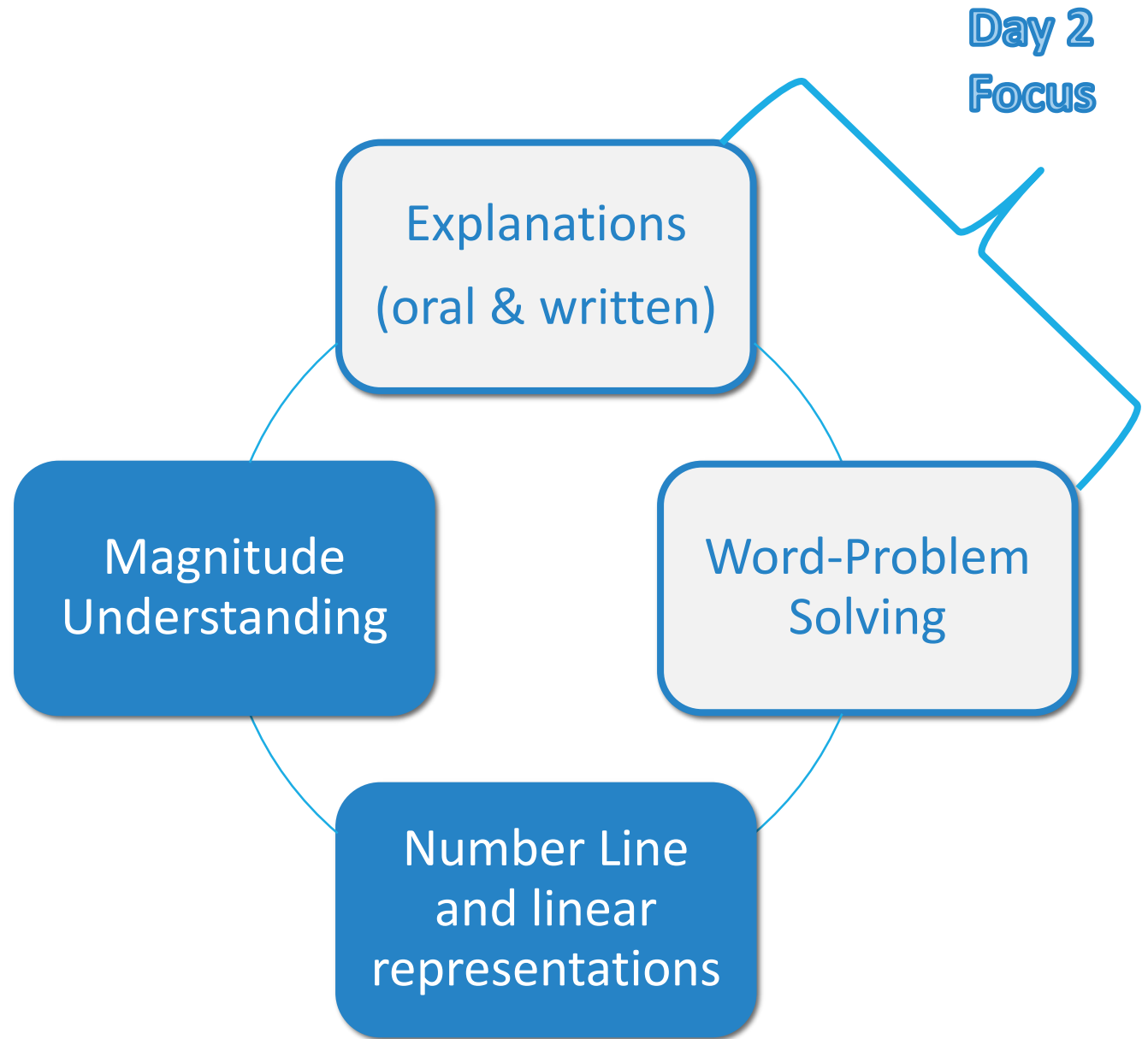
Lesson 4 Fidelity Sheet

Practice Pages

Concluding Reminders

Purpose of the Research Project

Big Ideas



Agenda

1. Review Homework, Lesson 36
2. Audio Recorders
3. Implementation Fidelity
4. Facilitating Verbal Explanations
5. Special Lessons
 - Performance Assessments
 - Written Explanations
6. “Extra Material” Demonstrations
7. Content Demonstrations, Clarifications, & Strategies
8. Word Problems
9. Practice
10. Concluding Reminders

Review Homework

Lesson 36

Go over Lesson 36

How did you do representing with both number lines and C-Rods?

Questions/comments about the lesson?

A lot of material not outlined in TransMath

- What are your questions?
- Take notes for this lesson down the road.

Discuss Lesson 37

Audio Recorders

Audio Recording Rules

Record all of your lessons

- Implementation fidelity
- Help troubleshoot

ALWAYS say the lesson # and group identifier before each lesson

“Lesson 4 Franklin Elementary, Group XX”

We will assign group numbers once they’re formed, so you’ll need to remember your group numbers.



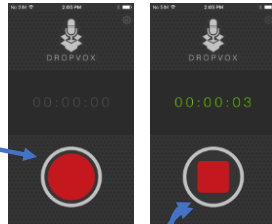
They will automatically upload to our dropbox folder

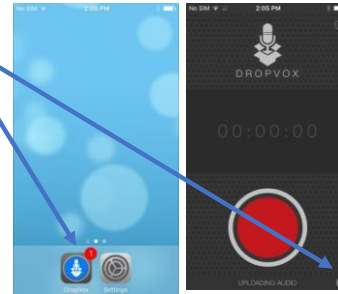
Make sure you are sufficiently charged before tutoring

Let’s look at your handout

iPhone Recorder Guide

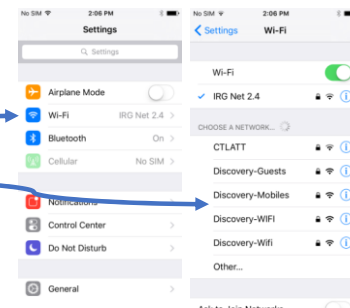
RECORDING

1. Make sure to charge the iPhone the night before your session.
2. Slide to Unlock 
3. Use DropVox App 
4. Press [RECORD] button to begin recording 
5. Begin recording by stating the date, session number, and other relevant information to identify the recording session.
6. Press [STOP] button **TWICE** to **PAUSE**
7. Press [STOP] button **ONCE** to **STOP**
8. Upon stopping the recording (by pressing [STOP] once), the iPhone will attempt to upload the files automatically to Dropbox when WiFi is available.
9. If there are numbers inside the circle from one of these screens, it means there are files that have not yet uploaded.
10. Find a Wifi signal and open the DropVox App to allow it to upload.



CONFIGURING WIFI

1. The iPhone needs to be connected to WiFi in order to upload. You can use the WiFi from your home, at a Starbucks, McDonalds, or other free WiFi/Hotspot locations.
2. Open settings, choose Wi-Fi, select the network and enter the password.



If you have any questions about the iPhone recorder or DropBox, please contact John:
Office: 714-826-9600 / Cell: 714-299-3961 / johnh@inresg.org

iPhone training video

<https://www.dropbox.com/s/nfw9fak6nngnt1p/iPhone%20Recorder%20Training%20Video.mov?dl=0>

Implementation Fidelity & CLASS observations

Implementation Fidelity

We will select lessons that include critical teaching elements

We do not divulge which lessons will include a fidelity check

We address observable behaviors

- Observed/not observed/not applicable
- Some you may need to narrate since we audio vs video lessons
- “I am drawing four number lines.”

We address quality of instruction

- Likert scale items

Experienced teachers and researchers will check fidelity

Lesson 4 sample on the following pages – also in your handouts

Trans Math Fraction Tutoring Fidelity Instrument
RCT Program 2016-2017
Lesson 4: Representing Fractions with Cuisenaire Rods

1

Rater Name: _____ Date: _____

Tutor Name: _____ School/District: _____

Review

| | | |
|---|---|---|
| 1. T provides time to do review activity. | Y | N |
|---|---|---|

Problem Solving: Representing Fractions with Cuisenaire Rods

Example 1: Teacher demonstration with Student Help

| | | |
|--|---|---|
| 1. T writes 1/5 on white board. | Y | N |
| 2. T asks students how the parts are named. | Y | N |
| 3. T and students asks and discusses how many 1/5 unit fractions make one whole. | Y | N |
| 4. T asks students to help show 1/5 with Cuisenaire Rods. | Y | N |
| 5. T says they need to show 1/5 next to one whole. | Y | N |
| 6. T prompts students to choose a red Cuisenaire rod for 1/5. | Y | N |
| 7. T asks students how many red Cuisenaire rods (1/5) they need to find length of one whole. | Y | N |
| 8. T and students line up 5 red Cuisenaire rods. | Y | N |
| 9. T and students try different Cuisenaire rods to see what equals one whole when red equals 1/5. [Listen for: 5 reds are the same length as 1 orange] | Y | N |
| 10. T and students determine orange is one whole by lining up 5 red and 1 orange. | Y | N |
| 11. T and students line up one red and one orange to show 1/5. | Y | N |
| 12. T shows students the visual in the materials for Example 1. | Y | N |
| 13. T reviews meaning of denominator. | Y | N |
| 14. T reviews meaning of numerator. | Y | N |
| 15. T reviews meaning of part. | Y | N |
| 16. T reviews meaning of denominator. | Y | N |

Example 2: Teacher demonstration with Student Help

| | | |
|--|---|---|
| 1. T reviews meaning of denominator. | Y | N |
| 2. T writes 2/5 on white board. | Y | N |
| 3. T and/or students write 2/5 on their white boards. | Y | N |
| 4. T asks students how 1/5 helps us show 2/5. | Y | N |
| 5. T shows 2/5 underneath 1/5 with C-Rods. | Y | N |
| 6. T and/or students write $2 \times 1/5 = 2/5$ | Y | N |
| 7. T shows students the visual in the materials for Example 2. | Y | N |

2

Check for Understanding: Student and teacher show fraction

| | | |
|---|---|---|
| 1. T asks students to write 3/5. | Y | N |
| 2. T asks students to show 3/5 with Cuisenaire rods on their own. | Y | N |
| 3. T and S discuss how to show 3/5 with Cuisenaire rods. | Y | N |
| 4. T and/or students write $3 \times 1/5 = 3/5$ | Y | N |

Problem Solving Activity (This portion of the lesson is more student-driven and less demonstration oriented.)

| | | | |
|--|---|---|-----|
| 1. T tells students to open student workbook to the appropriate page and | Y | N | |
| 2. T explains they will show these fractions with Cuisenaire rods on the white board. | Y | N | |
| 3. T reads the five fractions students will show. | Y | N | |
| 4. T assigns 2 specific fractions (one unit fraction and one non-unit fraction) to each student to model with Cuisenaire rods. | Y | N | |
| 5. T provides time for students to model their assigned fractions. | Y | N | |
| 6. T assists students in modeling their fractions, if needed. | Y | N | N/A |
| 7. T has student(s) explain 1/4. | Y | N | |
| 8. T provides corrective feedback during student explanations, if needed. | Y | N | N/A |
| 9. T asks prompting questions to help students explain, if needed. | Y | N | N/A |
| 10. T has student(s) explain 1/3. | Y | N | |
| 11. T provides corrective feedback during student explanations, if needed. | Y | N | N/A |
| 12. T asks prompting questions to help students explain, if needed. | Y | N | N/A |
| 13. T has student(s) explain 1/2. | Y | N | |
| 14. T provides corrective feedback during student explanations, if needed. | Y | N | N/A |
| 15. T asks prompting questions to help students explain, if needed. | Y | N | N/A |
| 16. T has student(s) explain 2/3. | Y | N | |
| 17. T provides corrective feedback during student explanations, if needed. | Y | N | N/A |
| 18. T asks prompting questions to help students explain, if needed. | Y | N | N/A |
| 19. T has student(s) explain 3/4. | Y | N | |
| 20. T provides corrective feedback during student explanations, if needed. | Y | N | N/A |
| 21. T asks prompting questions to help students explain, if needed. | Y | N | N/A |

Independent Work: Enter the amount of time in minutes and seconds that the tutor allocated to these activities. (:)

Min. Sec.

These are independent, silent practice activities unless students have questions. Students do not have to complete all three activities for a tutor to earn a Yes rating on this item.

Activity 1: Put fractions on a partitioned number line.

Activity 2: Dividing rectangles into fair shares.

Activity 3: Name the unit fraction represented by each pair of rods.

| | | | |
|---|---|---|-----|
| 1. T provides time for students to work on Independent Work problems. | Y | N | |
| 2. T provided assistance, if needed. | Y | N | N/A |

Summary Ratings

1. When students are explaining their answers, the tutor asks open-ended questions to probe thinking and understanding before providing an answer or solution.

Never Rarely Sometimes Frequently N/A

(1/2 ratings are acceptable)

2. Paces the lesson so that all parts of the session were covered in sufficient depth.

1 2 3 4 5
Low **Medium** **High**

3. Uses clear and mathematically correct language.

1 2 3 4 5
Low **Medium** **High**

4. Enhances students' explanations.

1 2 3 4 5
Low **Medium** **High**

5. Provides specific math-oriented praise.

1 2 3 4 5
Low **Medium** **High**

6. Maintains a positive rapport with students.

1 2 3 4 5
Low **Medium** **High**

7. Perception of students' grasp of the content.

1 2 3 4 5
Low **Medium** **High**

8. Overall rating of the tutor's implementation.

1 2 3 4 5
Low **Medium** **High**

CLASS

- You will be observed or video taped at least one time per group
- Some of you will be observed two times for reliability purposes
- We have outside staff trained as CLASS observers who will be viewing the lessons
- We will need to get video permission from all students in your tutoring group
 - more to come once we've assigned students to tutoring groups

Eliciting Verbal Explanations

Five types of Questions (See handout)

1. Initial eliciting of students' thinking
2. Probing students' answers
 - Trying to figure out what a student means or is thinking when you don't understand what they are saying
 - Checking whether right answers are supported by correct understanding
 - Probing wrong answers to understand student thinking
3. Supporting students to make connections (e.g., between a model and a mathematical idea or a specific notation)
4. Guiding students to reason mathematically (e.g., make conjectures, state definitions, generalize, prove)
5. Extending students' current thinking, and assessing how far they can be stretched

Our Major Focus is on the first two

1. Initial eliciting of students' thinking

2. Probing students' answers

- Trying to figure out what a student means or is thinking when you don't understand what they are saying
- Checking whether right answers are supported by correct understanding
- Probing wrong answers to understand student thinking

Videos

Embed Ball video:

Example of #1: initial eliciting of students thinking

Other videos where students are verbalizing what they wrote, written explanation section – we could discuss these types of questions

Special Lessons

- ❖ Performance Assessments
- ❖ Written Explanations

Performance Assessments

1. These are DATA and need to be returned to IRG
2. These are in your folder and are labeled with the PA number
3. These occur in five lessons:
 - Lesson 8: PA1
 - Lesson 18: PA2
 - Lesson 28: PA3
 - Lesson 38: PA4
 - Lesson 50: PA5
4. These are to be treated like an official test
 - No supporting materials can be used during Performance Assessments
 - No feedback given for Performance Assessments
5. Subsample of control students will take these so testing conditions need to be the SAME

Performance Assessments, PA1

IRG# _____ RCT_2016-2017

Performance Assessment

For the Student to Complete:

Last Name: _____ First Name: _____

Math Teacher Last Name: _____

School Name: _____ District: _____

For the Assessor to Complete:

Date: _____ Assessor Name: _____

Testing Condition (check one): ☐ Treatment ☐ Control

RCT_2016-2017

Mark where $\frac{2}{5}$ is located on the number line below.

0 6
5

How did you know where to place $\frac{2}{5}$ on the number line? Explain your thinking.

Performance Assessment Lesson 28



Review Vocabulary and Power Concepts

- Take only about 2-3 minutes
- Only go into depth if there seems to be a misunderstanding

?



Review

?

1. Activities 1-9

- Do this as a group review
- Go through as many as you can
- Save 20 minutes for the end to administer the Performance Assessment

?



Performance Assessment

- Give students PA3
- Do not help this is an assessment!
- Collect all papers for project staff

??

Explanation Lessons

Lesson 11: Introduce Activity: students observe while teacher models

Lesson 13

Lesson 18

Lesson 23

Lesson 27

Lesson 33

Lesson 37

Lesson 42

Lesson 46

WRITING EXPLANATIONS FOR MATH PROBLEMS

?

THINKING

1. What's the problem asking?

?

Compare, use a number line, draw a picture?
Add, subtract, multiply, divide?

?

2. What did I do to solve it?

?

Mark a number line?
Use fair shares?
Use benchmark fractions?
Rewrite fractions to get common denominators?

?

WRITING

3. Write all the steps.

- Remember to use mathematically correct vocabulary.

?

4. Write the answer and say why it makes sense.

With Examples

Without Examples

WRITING EXPLANATIONS FOR MATH PROBLEMS

?

THINKING

1. What's the problem asking?

?

2. What did I do to solve it?

?

WRITING

3. Write all the steps.

- Remember to use mathematically correct vocabulary.

?

4. Write the answer and say why it makes sense.

Written Explanations: Lesson 11

1. Tutor introduces the Explanation card (use the one with examples)
2. Tutor models solving the problem while students observe
3. Tutor solves correctly and uses a number line to show work
4. Tutor writes a complete explanation using at least 3 vocabulary words
5. Tutor writes an incomplete explanation

Day 11 Explanation Activity

Which fraction is greater? Write the $<$, $>$, or $=$.

$$\frac{4}{10} \text{ or } \frac{5}{6}$$

Explain how you solved the problem using at least 3 math vocabulary words from the vocabulary list.

Written Explanations: Lesson 11

Day 11 Explanation Activity – Answer Key

Which fraction is greater? Write the <, >, or =.

$$\frac{4}{10} \text{ or } \frac{5}{6}$$

$$\frac{4}{10} < \frac{5}{6}$$

“Tutor should draw a number lines and use the number line and benchmark numbers to solve it.”

Explain how you solved the problem using at least 3 math vocabulary words from the vocabulary list.

GOOD EXAMPLE: This is a compare problem. I need to compare the fractions and write which fraction is greater. The numerators are not the same and denominators are not the same. So I draw a number line to help me. I mark 0, $\frac{1}{2}$, and 1 for benchmark fractions to help compare. I start with $\frac{4}{10}$. I know it's less than $\frac{1}{2}$ because it's less than $\frac{5}{10}$. I put $\frac{4}{10}$ between 0 and $\frac{1}{2}$ on the number line. Now I look at $\frac{5}{6}$. I know it will be close to 1 because $\frac{5}{6}$ is almost $\frac{6}{6}$. I put it close to 1 on the number line. This shows me that $\frac{4}{10}$ is less than $\frac{5}{6}$.

“This is an example of one way to write a complete example. If the tutor wishes to modify this at all to suit their group understanding better, that is fine – just be sure to use at least 3 vocabulary words.”

BAD EXAMPLE: I looked at the numbers and I knew this one was bigger. I drew some pictures to help me. And that's all.

$$\frac{4}{10} < \frac{5}{6}$$

“Tutor should draw a number lines and use the number line and benchmark numbers to solve it.”

GOOD EXAMPLE: This is a compare problem. I need to compare the fractions and write which fraction is greater. The numerators are not the same and denominators are not the same. So I draw a number line to help me. I mark 0, $\frac{1}{2}$, and 1 for benchmark fractions to help compare. I start with $\frac{4}{10}$. I know it's less than $\frac{1}{2}$ because it's less than $\frac{5}{10}$. I put $\frac{4}{10}$ between 0 and $\frac{1}{2}$ on the number line. Now I look at $\frac{5}{6}$. I know it will be close to 1 because $\frac{5}{6}$ is almost $\frac{6}{6}$. I put it close to 1 on the number line. This shows me that $\frac{4}{10}$ is less than $\frac{5}{6}$.


“This is an example of one way to write a complete example. If the tutor wishes to modify this at all to suit their group understanding better, that is fine – just be sure to use at least 3 vocabulary words.”

BAD EXAMPLE: I looked at the numbers and I knew this one was bigger. I drew some pictures to help me. And that's all.

Written Explanations: Lesson 13

IRG Lesson 13 Written Explanation

Name _____ Date _____

 Written Explanation

Activity

Compare these two fractions. Explain your answer.

$\frac{2}{3}$ _____ $\frac{3}{4}$ _____

IRG Lesson 13 4


- Last activity in the Lesson
- Students use the Explanation Card (you can use with or without examples – tutor choice)
- Vocabulary List available
- Review the problem afterward and discuss complete answers

Written Explanations: Lesson 18

IRG – 18

Written Explanation

Name _____ Date _____

 **Written Explanation**

Activity _____

Place these four fractions in the correct categories.

1. $\frac{9}{8}, \frac{1}{4}, \frac{11}{12}, \frac{100}{99}$

Proper Fractions

Improper fractions

←

0

$\frac{1}{2}$

1

→


IRG • Lesson 18 1

- First activity in the Lesson
- Students use the Explanation Card
- Vocabulary List available
- Review the problem afterward and discuss complete answers

Written Explanations: Lesson 23

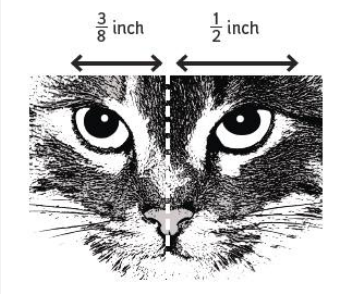
IRG 23 Written Explanation

Name _____ Date _____

 Written Explanation

Activity _____

This cat's face is supposed to be drawn proportionally, but the eyes are not exactly centered. The width of the cat's face from the left side to the center is $\frac{3}{8}$ inch and the width from the center to the right side is $\frac{1}{2}$ inch. What is the total width of the cat's face in the drawing?



IRG Lesson 23 1

- Third word problem within the activity (middle activity)
- Students use the Explanation Card
- Vocabulary List available
- Review the problem afterward and discuss complete answers

Written Explanation Videos

EMBED

Clip1_robin

Clip4a_robin

Clip4b_robin

Practice

Practice IRG-11 Explanation Activity


This is in your Practice Packet

Let's take 5 minutes to practice and then we'll share with the group.

IRG – 18

Written Explanation

Name _____ Date _____

 **Written Explanation**

Activity _____

Place these four fractions in the correct categories.

1. $\frac{9}{8}, \frac{1}{4}, \frac{11}{12}, \frac{100}{99}$

Proper Fractions

Improper fractions

←

0

$\frac{1}{2}$

1

→

IRG • Lesson 18

1

What are the differences between Explanations and Performance Assessments?

Which ones are considered tests?

Which ones are in your folder?

Which ones are in the student workbooks?

Which ones can you use explanation cards for?

Which ones do you send to IRG?

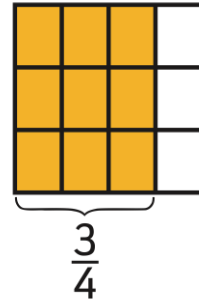
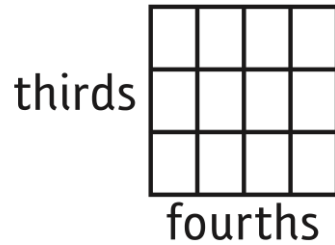
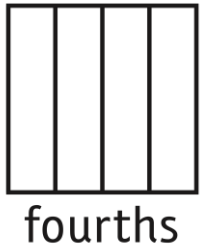
Which ones do you need the vocabulary list for?

Extra Material Demonstrations and Practice

Multiplication with Area Models

Introduced in Lesson 31

Use an area model to find $\frac{2}{3} \cdot \frac{3}{4}$.



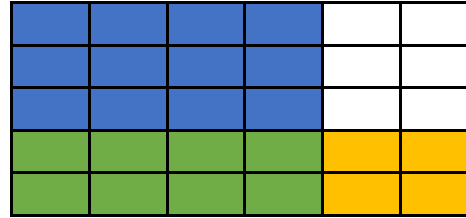
These are the steps in TransMath

Area models are encouraged before we teach the standard algorithm

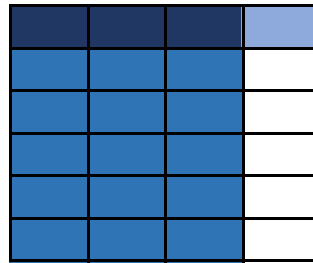
Multiplication with Area Models

$$\frac{1}{2} \times \frac{1}{2}$$

Model $\frac{2}{5} \times \frac{5}{6}$



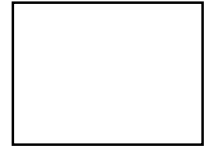
Model $\frac{1}{6} \times \frac{3}{4}$



What are some ways to show overlap when you only have one color?

- Diagonal lines
- Stripes vs. shade

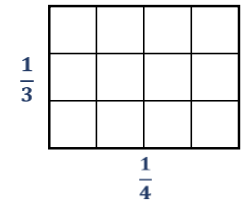
Draw a rectangle.



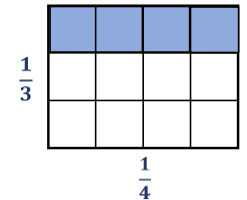
Write the first fraction on the side. Divide the rectangle by the number in the denominator.



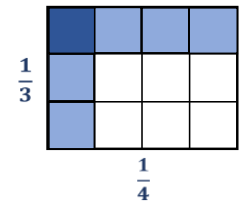
Write the second fraction on the bottom. Divide the rectangle by the number in the denominator.



Lightly shade the numerator for the first fraction.



Lightly shade the numerator for the second fraction.



Multiplication Answer:
Shaded overlap = Numerator
Total regions in the rectangle = Denominator

} $\frac{!}{!} \%$

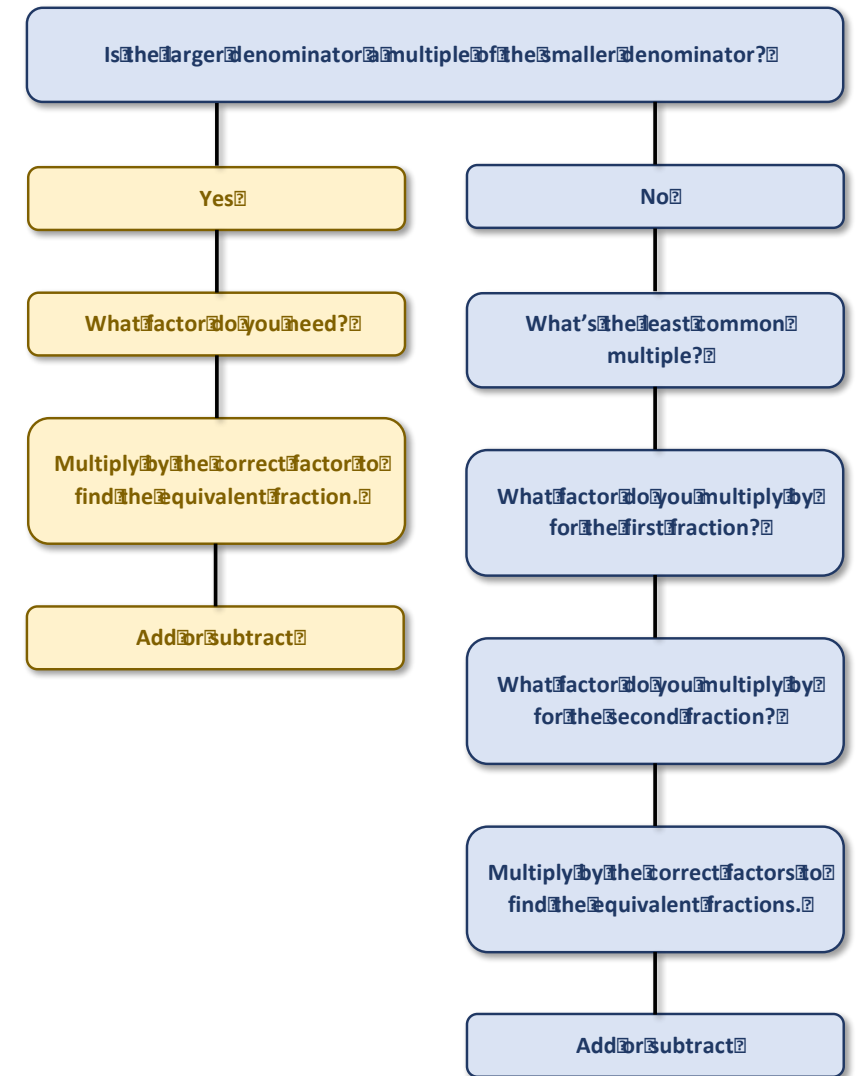
Multiplication with area model video

Embed clip2_robin

Adding and Subtracting

- Introduce the yellow side at the end of Lesson 22
- Introduce the blue side at the end of Lesson 24
 - Common multiples (multiply denominators together)
 - Least common multiples in Lesson 26
- Discuss three problem types for addition and subtraction in Lesson 25
 - when the card ends up being most necessary for using decision rules

Finding a Common Denominator for Fraction Addition and Subtraction



Adding and Subtracting

From Lesson 22

Find the difference: $\frac{7}{8} - \frac{1}{4}$.

Begin by using multiplication to find an equivalent fraction for $\frac{1}{4}$. The equivalent fraction needs to have fair shares of eighths because the other fraction in the problem has fair shares of eighths.

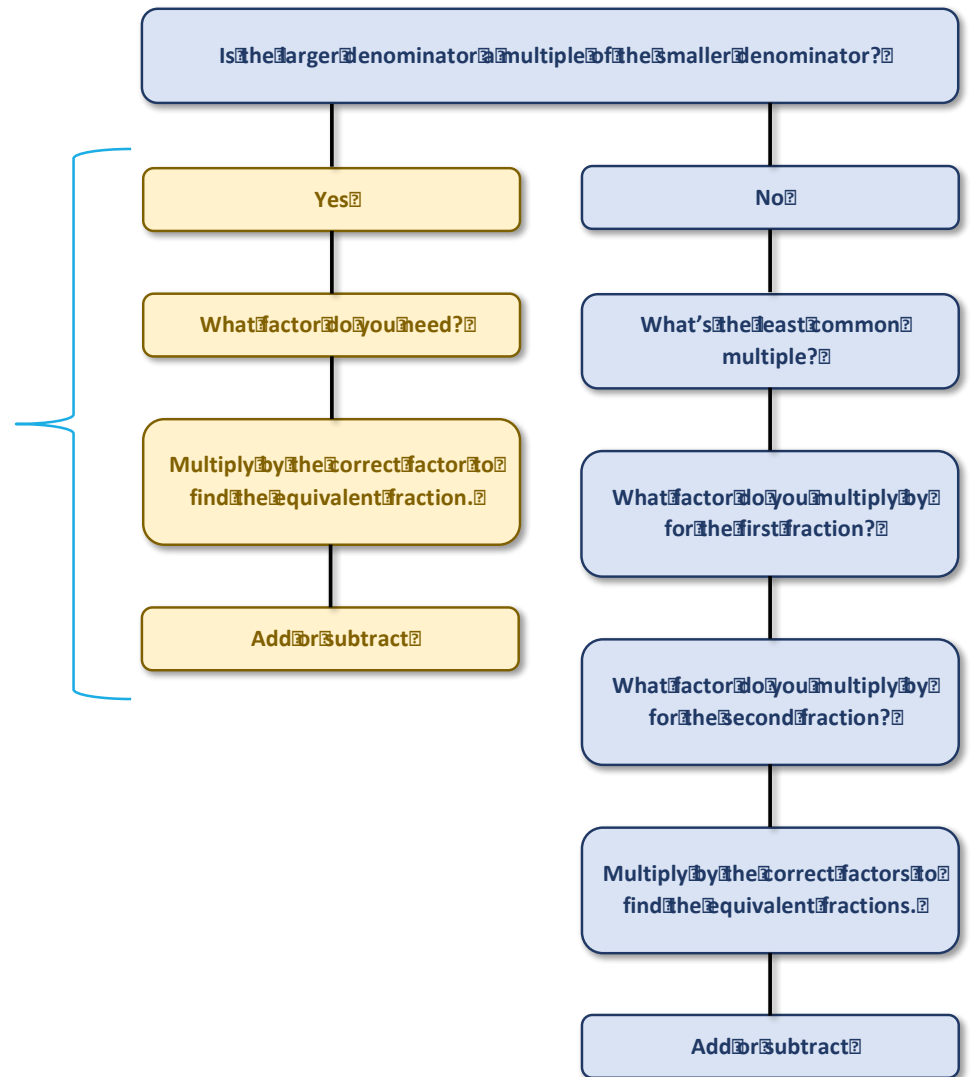
$$\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$$

Now we can substitute $\frac{2}{8}$ for $\frac{1}{4}$ in the problem and subtract fair shares.

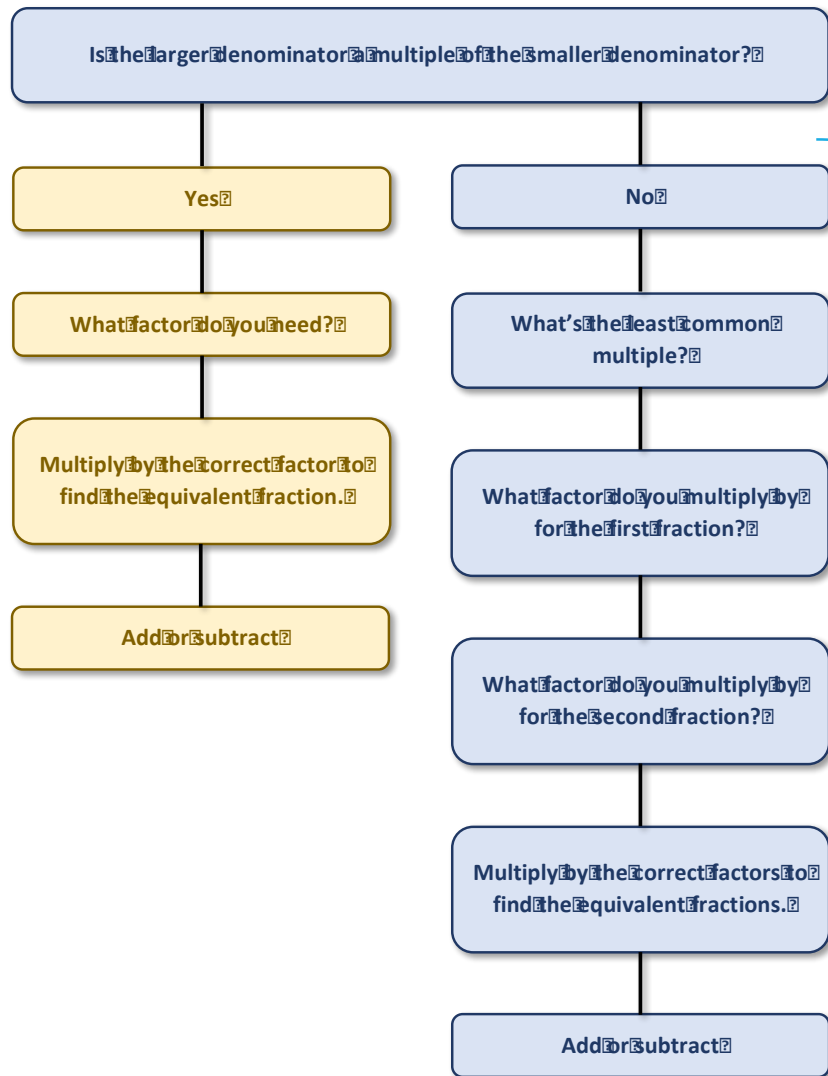
$$\begin{array}{r} \frac{2}{8} \\ \downarrow \\ \frac{7}{8} - \frac{1}{4} = \frac{5}{8} \end{array}$$

$$\text{So, } \frac{7}{8} - \frac{1}{4} = \frac{5}{8}.$$

Finding a Common Denominator for Fraction Addition and Subtraction



Finding a Common Denominator for Fraction Addition and Subtraction



Adding and Subtracting

From Lesson 24

Find $\frac{1}{2} + \frac{1}{5}$ by using the common denominator 10.

Because the denominators are different and not multiples of one another, both fractions need to be rewritten. Use the common denominator 10 to write equivalent fractions.

Begin with $\frac{1}{2}$. To get from a denominator of 2 to a denominator of 10, multiply 2 by 5. Remember to multiply the numerator by the same number.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

Now write the equivalent fraction for $\frac{1}{5}$. To get from a denominator of 5 to a denominator of 10, multiply 5 by 2. Remember to multiply the numerator by the same number.

$$\frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}$$

Now add the equivalent fractions.

$$\frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

LAPS: Mixed Number Addition & Subtraction

L—LOOK at the problem carefully.

- Make sure that the numbers are lined up.
- Decide if addition or subtraction is supposed to be performed.

$$\begin{array}{r} 3\frac{1}{5} \\ + 2\frac{3}{5} \\ \hline \end{array}$$

In this problem, the fractions and whole numbers are lined up, and we need to add.

A—ALTER the problem if necessary.

Alter means change. Sometimes we need to change something about a problem before we solve it. When adding or subtracting fractions, the denominators need to be the same.

$$\begin{array}{r} 3\frac{1}{5} \\ + 2\frac{3}{5} \\ \hline \end{array}$$

In this problem, the denominators are the same, so we don't have to alter the fractions.

P—PERFORM the operation.

Now we are ready to add. We begin by adding the fractional parts of the two numbers.

Next we add the whole numbers.

$$\begin{array}{r} 3\frac{1}{5} \\ + 2\frac{3}{5} \\ \hline 5\frac{4}{5} \end{array}$$

S—SIMPLIFY the answer.

We have the answer we want because the answer is a mixed number in simplest form. So, for this problem, we do not need to do anything in this step.

L—LOOK at the problem carefully.

- Make sure the numbers are lined up.
- Decide if addition or subtraction is supposed to be performed.

$$\begin{array}{r} 5\frac{2}{3} \\ - 1\frac{1}{3} \\ \hline \end{array}$$

In this problem, the fractions and whole numbers are lined up, and we need to subtract.

A—ALTER the problem if necessary.

Decide if the denominators need to change.

$$\begin{array}{r} 5\frac{2}{3} \\ - 1\frac{1}{3} \\ \hline \end{array}$$

The denominators are the same, so we do not need to change anything before we subtract these two numbers.

P—PERFORM the operation.

Now we do the subtraction.

First we subtract the fractional parts.

Then we subtract the whole numbers.

$$\begin{array}{r} 5\frac{2}{3} \\ - 1\frac{1}{3} \\ \hline 4\frac{1}{3} \end{array}$$

S—SIMPLIFY the answer.

We have the answer we want because the answer is a mixed number in simplest form. So, for this problem, we do not need to do anything in this step.

LAPS: Mixed Number Addition & Subtraction

L.A.P.S.

Look

Are common denominators needed?
Is regrouping needed?

Alter

Change denominators
Regroup

Perform

Add or subtract

Simplify

Reduce
Regroup

LAPS: Video

Clip3_Pat

Clip4_Pat

Regrouping with LAPS

Lesson 47:

Addition: regroup to simplify

$$\begin{array}{r} 2\frac{2}{3} \\ + 1\frac{2}{3} \\ \hline 4\frac{3}{3} \end{array}$$

$$\begin{array}{r|l} 2 & \frac{2}{3} \\ + & 1\frac{2}{3} \\ \hline & \frac{3}{3} + \frac{1}{3} \end{array}$$

$$\begin{array}{r|l} 1 & \frac{2}{3} \\ 2 & \frac{2}{3} \\ + & 1\frac{2}{3} \\ \hline 4 & \frac{1}{3} \end{array}$$

Our answer is $4\frac{1}{3}$.

$$\begin{array}{r} 2\frac{2}{3} \\ + 1\frac{2}{3} \\ \hline 3\frac{4}{3} \end{array}$$

$$\begin{array}{r|l} \frac{3}{3} \text{ or } 1 & \frac{2}{3} \\ 2 & \frac{2}{3} \\ + & 1\frac{2}{3} \\ \hline & \frac{3}{3} + \frac{1}{3} \end{array}$$

L.A.P.S.

Look

Are common denominators needed?
Is regrouping needed?

Alter

Change denominators
Regroup

Perform

Add or subtract

Simplify

Reduce
Regroup

Regrouping with LAPS

Lesson 47:

Subtraction: regroup to alter

$$\begin{array}{r|l} 3 & \frac{2}{5} \\ - 1 & \frac{4}{5} \\ \hline \end{array}$$

$$\begin{array}{r|l} 2 & \frac{2}{5} + \frac{5}{5} \\ - 1 & \frac{4}{5} \\ \hline \end{array}$$

$$\begin{array}{r|l} 2 & \frac{7}{5} \\ - 1 & \frac{4}{5} \\ \hline & \frac{3}{5} \end{array}$$

$$\begin{array}{r|l} 2 + \frac{5}{5} & \\ \cancel{3} & \frac{2}{5} \\ - 1 & \frac{4}{5} \\ \hline \end{array}$$

$$\begin{array}{r|l} 2 & \frac{7}{5} \\ - 1 & \frac{4}{5} \\ \hline \end{array}$$

$$\begin{array}{r|l} 2 & \frac{7}{5} \\ - 1 & \frac{4}{5} \\ \hline 1 & \frac{3}{5} \end{array}$$

L.A.P.S.

Look

Are common denominators needed?
Is regrouping needed?

Alter

Change denominators
Regroup

Perform

Add or subtract

Simplify

Reduce
Regroup

Regrouping with LAPS

Lesson 48:

Addition: reduce to simplify answer

$$\begin{array}{r} \boxed{\text{L}} \\ 2\frac{1}{2} \\ + 3\frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} \boxed{\text{A}} \\ \frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6} \\ \\ 2\frac{3}{6} \\ + 3\frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} \boxed{\text{P}} \\ 2\frac{3}{6} \\ + 3\frac{1}{6} \\ \hline 5\frac{4}{6} \end{array}$$

$$\begin{array}{r} \boxed{\text{S}} \\ \frac{4}{6} = \frac{2}{2} \cdot \frac{2}{3} \\ \\ 1 \cdot \frac{2}{3} = \frac{2}{3} \\ 5\frac{4}{6} = 5\frac{2}{3} \end{array}$$

$$2\frac{1}{2} + 3\frac{1}{6} = 5\frac{2}{3}$$

L.A.P.S.

Look

Are common denominators needed?
Is regrouping needed?

Alter

Change denominators
Regroup

Perform

Add or subtract

Simplify

Reduce
Regroup

Practice LAPS

$$1\frac{2}{5} + 3\frac{4}{5} =$$

$$3\frac{6}{8} - 2\frac{5}{8} =$$

$$4\frac{1}{6} - 2\frac{1}{2} =$$

$$3\frac{3}{4} + 1\frac{3}{4} =$$

Practice LAPS

$$1\frac{2}{5} + 3\frac{4}{5} = 4\frac{6}{5} = 5\frac{1}{5}$$

$$3\frac{6}{8} - 2\frac{5}{8} = 1\frac{1}{8}$$

$$4\frac{1}{6} - 2\frac{1}{2} = 4\frac{1}{6} - 2\frac{3}{6} = 3\frac{7}{6} - 2\frac{3}{6} =$$

$$1\frac{4}{6} = 1\frac{2}{3}$$

$$3\frac{3}{4} + 1\frac{3}{4} = 4\frac{6}{4}$$

$$4\frac{2}{3}$$

Algorithms for the four Operations

Summary of Rules for Operations with Fractions

Addition

We need a common denominator. Once we have a common denominator, we add the numerators and leave the denominator the same. Simplify the answer.

Subtraction

We need a common denominator. Once we have a common denominator, we subtract the numerators and leave the denominator the same. Simplify the answer.

Multiplication

We multiply across, numerator times numerator and denominator times denominator. Simplify the answer.

Division

Invert and multiply. Simplify the answer.

Practice Algorithms for the four Operations

$$\frac{2}{5} + \frac{4}{5} =$$

$$\frac{6}{8} - \frac{5}{8} =$$

$$\frac{1}{2} \div 3 =$$

$$\frac{1}{6} + \frac{4}{6} =$$

$$\frac{6}{4} - \frac{3}{4} =$$

$$\frac{1}{8} + \frac{3}{4} =$$

$$\frac{6}{8} - \frac{1}{2} =$$

$$\frac{3}{4} \div \frac{1}{4} =$$

$$\frac{3}{4} - \frac{1}{8} =$$

$$\frac{1}{4} \times \frac{4}{10} =$$

$$\frac{1}{2} \times \frac{6}{8} =$$

$$2 \div \frac{1}{4} =$$

Algorithms for the four Operations

$$\frac{2}{5} + \frac{4}{5} = \frac{6}{5}$$

$$\frac{6}{8} - \frac{5}{8} = \frac{3}{8}$$

$$\frac{1}{2} \div 3 = \frac{1}{6}$$

$$\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$$

$$\frac{6}{4} - \frac{3}{4} = \frac{1}{4}$$

$$\frac{1}{8} + \frac{3}{4} = \frac{7}{8}$$

$$\frac{6}{8} - \frac{1}{2} = \frac{2}{8}$$

$$\frac{3}{4} \div \frac{1}{4} = 3$$

$$\frac{3}{4} - \frac{1}{8} = \frac{5}{8}$$

$$\frac{1}{4} \times \frac{4}{10} = \frac{4}{40} = \frac{1}{10}$$

$$\frac{1}{2} \times \frac{6}{8} = \frac{6}{16} = \frac{3}{8}$$

$$2 \div \frac{1}{4} = 8$$

Content Demonstrations, Clarifications, & Strategies

Multiple vs.

Factor

MULTIPLE

- A bigger number.
- Multiple is the product when you multiply a whole number by another number

Example: 2: 2, 4, 6, 8, 10, 12

Example: 6: 6, 12, 18, 24

All the numbers listed are the multiples.

This list can go infinitely.

FACTOR

- A smaller number.
- Factor is a number that can be divided into a number without a remainder.

Example: 2: 1, 2

Example: 6: 1, 2, 3, 6

All the numbers listed are the factors

Common Multiple vs.

COMMON MULTIPLE

- A bigger number.
- Common Multiple is when a two different whole numbers can be multiplied by a whole number and they have the same product.

Example: 2: 2, 4, 6, 8, 10, 12

Example: 6: 6, 12, 18, 24

What are the common multiples?

6, 12, 18, 24

Common Factor

COMMON FACTOR

- A smaller number.
- Common Factor is a number that can be divided into a number without a remainder and it's common when two numbers have the same factor.

Example: 2: 1, 2

Example: 6: 1, 2, 3, 6

What are the common factors?

1, 2

Least Common Multiple vs. Greatest Common Factor

LEAST COMMON MULTIPLE

- A bigger number.
- Least Common Multiple is used for finding a common denominator

Example: 2: 2, 4, 6, 8, 10, 12

Example: 6: 6, 12, 18, 24

What's the least common multiple?

6

GREATEST COMMON FACTOR

- A smaller number.
- Greatest Common Factor is used for simplifying your answer.

Example: 2: 1, 2

Example: 6: 1, 2, 3, 6

What's the greatest common factor?

2

Showing Common Multiples: Video

Embed Clip3_robin

What does it mean to Commute?

Lesson 39

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{3 \times 4} = \frac{3}{3} \times \frac{2}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{6}{9} \times \frac{3}{5} = \frac{6 \times 3}{9 \times 5} = \frac{3 \times 2 \times 2}{3 \times 3 \times 5} = \frac{2 \times 2}{3 \times 5} = \frac{4}{15}$$

Try some: $\frac{4}{5} \times \frac{1}{4} =$

$$\frac{4}{5} \times \frac{5}{8} =$$

How is commuting like simplifying answers?

Error Analysis Lesson 41

Improve Your Skills

It is easy to make a mistake if we forget what operation we are working with.

$$\frac{3}{5} \cdot \frac{3}{6}$$

To solve a multiplication problem, begin by multiplying across. If we forget what operation we are working with, we might invert and multiply.

$$\frac{3}{5} \cdot \frac{3}{6} = \frac{3 \cdot 6}{5 \cdot 3} = \frac{18}{15}$$

ERROR

To check that the answer is correct, compare the answer to the factors. Remember, when we take a fraction of a fraction, we usually end up with a number that is less than one or both of the factors. In terms of fractions, $\frac{18}{15}$ is not less than either factor.

It is greater than 1 because $\frac{15}{15}$ is 1, and there are $\frac{3}{15}$ left over. This should clue us in that our answer is not correct.

When we solve the problem the correct way, the answer looks like this:

$$\frac{3}{5} \cdot \frac{3}{6} = \frac{9}{30}$$

CORRECT

To simplify the answer, we pull out the GCF, 3:

$$\frac{9}{30} = \frac{3}{3} \cdot \frac{3}{10} = 1 \cdot \frac{3}{10}$$

The simplified answer is $\frac{3}{10}$.

This is much less than $\frac{18}{15}$. The fraction $\frac{3}{10}$ is closer to 0 on the number line than 1. Here is where the numbers are located on a number line.



It's important to think about how the product compares to the factors.

Improve Your Skills

Here is a division problem.

$$\frac{4}{5} \div \frac{1}{5}$$

Suppose we use the algorithm for multiplication, which is to multiply across in the first step.

$$\frac{4}{5} \div \frac{1}{5} = \frac{4 \cdot 1}{5 \cdot 5} = \frac{4}{25}$$

ERROR

This answer is much less than the dividend. The answer to a division problem involving proper fractions should be greater than the dividend. How many groups of $\frac{1}{5}$ are there in $\frac{4}{5}$? Let's compare $\frac{4}{25}$ to the correct answer.

$$\frac{4}{5} \div \frac{1}{5} = \frac{4}{5} \cdot \frac{5}{1} = \frac{4 \cdot 5}{5 \cdot 1} = \frac{20}{5}$$

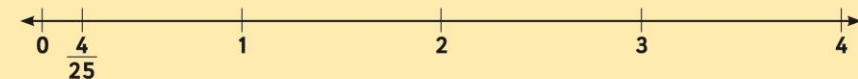
CORRECT

To simplify this answer, we pull out the GCF of 5:

$$\begin{aligned} \frac{20}{5} &= \frac{5}{5} \cdot \frac{4}{1} \\ &= 1 \cdot \frac{4}{1} \\ &= 4 \end{aligned}$$

There are four groups of $\frac{1}{5}$ in the number $\frac{4}{5}$.

The answer should be 4, which is quite a bit greater than $\frac{4}{25}$. We can see this on a number line.



Error Analysis Lesson 41

Improve Your Skills

Adding or Subtracting Across

Sometimes we might forget that we do not "add across" or "subtract across." Here is what this error looks like.

Addition

$$\frac{3}{5} + \frac{1}{4}$$

$$\frac{3+1}{5+4} = \frac{4}{9} \quad \text{ERROR}$$

This answer is incorrect.



Correct Addition

$$\begin{aligned} \frac{3}{5} + \frac{1}{4} &= \frac{12}{20} + \frac{5}{20} \\ &= \frac{17}{20} \end{aligned}$$

Subtraction

$$\frac{3}{5} - \frac{1}{4}$$

$$\frac{3-1}{5-4} = \frac{2}{1} \quad \text{ERROR}$$

This answer is incorrect.



Correct Subtraction

$$\begin{aligned} \frac{3}{5} - \frac{1}{4} &= \frac{12}{20} - \frac{5}{20} \\ &= \frac{7}{20} \end{aligned}$$

Strategies

Part to whole and whole to part

Partitioning odd number of parts

Fraction Patterns with number lines; Same D vs Same N

Strategies for reasoning about magnitude

- Relative size
- Benchmark numbers
- Proper vs improper strategies

Determining which number to put first in subtraction word problems

Word Problems

Word Problems

1. Some lessons are devoted only to word problems
2. Some lessons have 2-4 word problems included
3. Word problems address a concept covered that day or in previous days
4. Typically include the four operations
5. Includes mixed number addition and subtraction (LAPS)
6. Sometimes included as an Explanation Activities

Word Problem Lessons

Lesson 23: [Word Problem with explanation activity embedded to #3](#)

Lesson 27

Lesson 32: [Look at word problem with \$\frac{3}{4}\$ in it twice; discuss meaning](#)

Lesson 40: [modeling correct multiplication and division](#)

Lesson 43

Lesson 46

Lesson 23: taken from two old TransMath Lessons

Problem-Solving Activity

(Interactive Text, pages 78–79)

Have students turn to pages 78 and 79 in the *Interactive Text*, which provide an opportunity to practice solving word problems with fraction addition. There are four problems involving addition of fractions where one of the fractions needs to be changed to a common denominator.

Monitor students' work as they complete these tasks. Ask students to explain their thinking.

Watch for:

- Can students identify the fraction addition problem they are to solve?
- Can students model the problem or draw a picture to reason about what the problem is asking for?
- Can students find the common denominator by one of the methods they have learned for finding an equivalent fraction?
- Can students add the fractions accurately and come up with the correct answer?

Lesson 8 Problem-Solving Activity

Name _____ Date _____

Problem-Solving Activity
Fraction Addition in Word Problems

Solve the following word problems using Cuisenaire rods. Then check your answer using multiplication to make equivalent fractions.

1. Hector works for the city, and he puts pipes under roads. The pipes carry water from rain. He and the other workers dig a trench, put in the pipe, cover it with dirt, and pave the road. The pipe is $\frac{1}{4}$ unit wide and the dirt is $\frac{1}{2}$ unit deep. How many units is it from the bottom of the pipe to the street?

$\frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$. It is $\frac{3}{4}$ unit from the bottom of the pipe to the street.

2. A drawing of a house is shown. When the length of the front of the house is measured, it is $\frac{6}{10}$ unit long. Each side of the house measures $\frac{2}{5}$ unit long. What is the combined length of the front and two sides of the house in the drawing?

$\frac{6}{10} + \frac{2}{5} + \frac{2}{5} = \frac{6}{10} + \frac{4}{10} + \frac{4}{10} = \frac{14}{10}$. The combined length is $\frac{14}{10}$ units.

78 Unit 2 • Lesson 8

Lesson 8 Problem-Solving Activity

Name _____ Date _____

Problem-Solving Activity

3. This cat's face is supposed to be drawn proportionally, but the eyes are not exactly centered. The width of the cat's face from the left side to the center is $\frac{3}{8}$ inch and the width from the center to the right side is $\frac{1}{2}$ inch. What is the total width of the cat's face in the drawing?

$\frac{3}{8} + \frac{1}{2} = \frac{3}{8} + \frac{4}{8} = \frac{7}{8}$. The total width is $\frac{7}{8}$ inch.

4. This drawing shows an old human skull. A scientist wants to know the total distance from the back of the head to the front teeth. The scientist makes the measurements shown. How many units is it from the back of the head to the front teeth?

$\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} = \frac{5}{4}$. It is $\frac{5}{4}$ units from the back of the head to the front teeth.

79 Unit 2 • Lesson 8

Lesson 9

Problem-Solving Activity

(Interactive Text, pages 83–84)

Have students turn to pages 83 and 84 in the *Interactive Text*, which provide an opportunity to practice solving word problems with fraction subtraction. There are three problems involving subtraction of fractions where one of the fractions needs to be changed to the common denominator.

Monitor students' work as they complete these tasks. Ask various students to explain their thinking as you circulate around the room.

Watch for:

- Can students identify the fraction subtraction problem they are to solve?
- Can students model the problem or draw a picture to reason about what the problem is asking for?
- Can students find the common denominator by one of the methods they have learned for finding an equivalent fraction?
- Can students subtract the fractions accurately and come up with the correct answer?

Reinforce Understanding
Remind students that they can review lesson concepts by accessing the online *Unit 2 Lesson 9 Problem Solving Teacher Talk Tutorial*.

Lesson 9 Problem-Solving Activity

Name _____ Date _____

Problem-Solving Activity
Word Problems Using Fraction Subtraction

Use Cuisenaire rods to solve the following word problems. Then check your answer using multiplication to make equivalent fractions.

All the problems below involve Frieda. Every day when she rides home on the bus, she draws pictures of monsters.

1. What is the difference between the height of the head and the length of the tail?

$\frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{4}{8} = \frac{3}{8}$. The difference between the height of the head and the length of the tail is $\frac{3}{8}$ unit.

Unit 2 • Lesson 9 83

Lesson 9 Problem-Solving Activity

Name _____ Date _____

Problem-Solving Activity

2. What is the difference between the length and the width of this drawing?

$2\frac{1}{4} - 1\frac{1}{4} = 1$. The difference between the length and the width of the drawing is 1 unit.

3. What is the difference between the width of the fist and the height of the head?

$\frac{6}{10} - \frac{5}{10} = \frac{1}{10}$. The difference between the width of the fist and the height of the head is $\frac{1}{10}$ unit.

Reinforce Understanding
Use the Unit 2 Lesson 9 Problem Solving Teacher Talk Tutorial to review lesson concepts.

84 Unit 2 • Lesson 9

Lesson 23

IRG – 23 Written Explanation

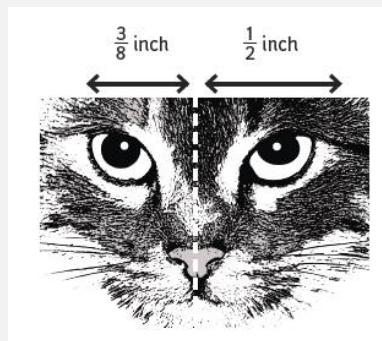
Name _____ Date _____



Written Explanation

Activity

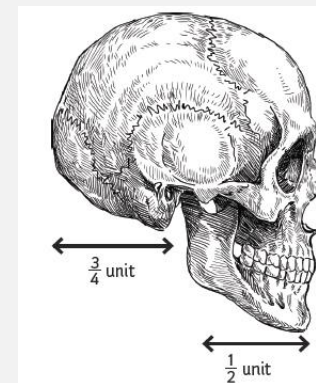
This cat's face is supposed to be drawn proportionally, but the eyes are not exactly centered. The width of the cat's face from the left side to the center is $\frac{3}{8}$ inch and the width from the center to the right side is $\frac{1}{2}$ inch. What is the total width of the cat's face in the drawing?



IRG – 23 Problem Solving/Group Work (cont.)

Name _____ Date _____

- This drawing shows an old human skull. A scientist wants to know the total distance from the back of the head to the front teeth. The scientist makes the measurements shown. How many units is it from the back of the head to the front teeth?



Lesson 32: Teacher Materials & Student Text

Lesson 3

Problem Solving: ► Multiplication Word Problems Using Area Models

How can we use an area model to help us solve word problems?
(Student Text, page 169)

Demonstrate

- Have students turn to page 169 of the *Student Text*. Explain to them that sometimes an area model provides a good visual for a real-world problem involving multiplication. Students are encouraged to use an area model in today's activity. Tell them that area models can be used to represent the drawings for floor plans or store layouts like the one shown in the diagram on this page.
- Have a volunteer read the problem about the store owner and the size of the candy section in the store. Discuss with students the pertinent information and ask a volunteer to write the multiplication problem that would solve the problem on the board. Re-read the problem if necessary.
- Next have students look at the area model. Ask them to describe the different sections of the model and what they represent.

Listen for:

- The area model is divided into fourths with vertical lines.
- The area model is divided into halves with a horizontal line.
- The shaded vertical section shows the fraction $\frac{1}{4}$.
- The shaded horizontal section shows the fraction $\frac{1}{2}$.

276 Unit 3 • Lesson 3

Lesson 3

► Problem Solving: Multiplication Word Problems Using Area Models

How can we use an area model to help us solve word problems?

Stores at gas stations are divided into different sections based on what is being sold. There are sections for items like candy, snacks, beverages, and hot food. The drawing shows the layout of a new store.



The owner wants $\frac{1}{2}$ of the store to be sodas, milk, candy, and snacks. The candy section should be $\frac{1}{4}$ of this section. What fraction of the entire store is the candy section?

Use an area model to answer this question because the map of the store and an area model look a lot alike. An area model is a good tool for visualizing this problem.

To solve the problem, we need to find $\frac{1}{4}$ of $\frac{1}{2}$. The drawing shows the model divided into halves. Complete the model by showing fourths in the other direction. Then shade to show $\frac{1}{4}$ and $\frac{1}{2}$. The product of $\frac{1}{4}$ and $\frac{1}{2}$, or $\frac{1}{8}$ of $\frac{1}{2}$, is shown by the overlapping shaded section.



There are eight fair shares in the model. The shading overlaps in one of the eight fair shares, the candy section. So $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$. This means that the candy section is one-eighth of the entire store.

Again, notice that the product is less than the factors. The area model shows clearly that $\frac{1}{8}$ is less than the factors $\frac{1}{2}$ and $\frac{1}{4}$.

Problem-Solving Activity
Turn to *Interactive Text*,
page 118.

Reinforce Understanding
Use the *Unit 3 Lesson 3 Problem Solving Teacher Talk*
Tutorial to review lesson concepts.

Unit 3 • Lesson 3 169

- The area where the shading for $\frac{1}{4}$ and $\frac{1}{2}$ overlap is the product, $\frac{1}{8}$. There are 8 total sections and 1 is shaded from both fractions.

- As one last final reminder, point out that the answer $\frac{1}{8}$ is less than $\frac{1}{4}$. Make sure students realize that this is because we are finding a fraction of a fraction.

Reinforce Understanding

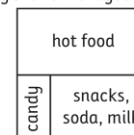
Remind students that they can review lesson concepts by accessing the online *Unit 3 Lesson 3 Problem Solving Teacher Talk Tutorial*.

Lesson 3

► Problem Solving: Multiplication Word Problems Using Area Models

How can we use an area model to help us solve word problems?

Stores at gas stations are divided into different sections based on what is being sold. There are sections for items like candy, snacks, beverages, and hot food. The drawing shows the layout of a new store.



The owner wants $\frac{1}{2}$ of the store to be sodas, milk, candy, and snacks. The candy section should be $\frac{1}{4}$ of this section. What fraction of the entire store is the candy section?

Use an area model to answer this question because the map of the store and an area model look a lot alike. An area model is a good tool for visualizing this problem.

To solve the problem, we need to find $\frac{1}{4}$ of $\frac{1}{2}$. The drawing shows the model divided into halves. Complete the model by showing fourths in the other direction. Then shade to show $\frac{1}{2}$ and $\frac{1}{4}$. The product of $\frac{1}{4}$ and $\frac{1}{2}$, or $\frac{1}{8}$ of $\frac{1}{2}$, is shown by the overlapping shaded section.



There are eight fair shares in the model. The shading overlaps in one of the eight fair shares, the candy section. So $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$. This means that the candy section is one-eighth of the entire store.

Again, notice that the product is less than the factors. The area model shows clearly that $\frac{1}{8}$ is less than the factors $\frac{1}{2}$ and $\frac{1}{4}$.

Problem-Solving Activity
Turn to *Interactive Text*,
page 118.

Reinforce Understanding
Use the *Unit 3 Lesson 3 Problem Solving Teacher Talk*
Tutorial to review lesson concepts.

Unit 3 • Lesson 3 169

Lesson 32: Teacher Materials - Answer Key

Problem-Solving Activity (Interactive Text, pages 118–119)

Have students turn to pages 118 and 119 of the *Interactive Text*. Tell them they will now have a chance to solve word problems using area models. This activity is about a company that replaces cracked and broken glass windows. The company's repair crew goes to a house and cuts the proper size of glass from a full sheet of glass. Tell students to imagine that the full sheet of glass is 1 unit by 1 unit square so that they can represent the problems with an area model.

Monitor students' work as they complete this activity.

Watch for:

- Can students divide the area model into the appropriate fractional parts?
- Can students shade the parts of the area model to correctly represent the fractions in the problems?
- Do students recognize that the overlap of the shaded sections represents the product?
- Can students determine the correct answer by counting the total parts for the denominator and counting the overlapping shaded parts for the numerator?

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *Unit 3 Lesson 3 Problem Solving Teacher Talk Tutorial*.

Lesson 3
Problem-Solving Activity


Name _____ Date _____

Problem-Solving Activity


Word Problems

Solve the following word problems. Use area models to help you solve each problem.

Holtom Glass Company replaces cracked and broken glass windows. When the repairers arrive at a house, they cut the glass for the windows from a large sheet of glass like the one shown. The repair crew has windows to fix at two different houses today.




1. At the first house, the repair crew has to fix two windows. For one window, they cut the sheet of glass in $\frac{1}{2}$ and then cut $\frac{2}{3}$ from one of the halves. What fraction of the full sheet do they use for this window?



$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$ Two-sixths of the full sheet of glass was used.

2. They take the other half of the full sheet of glass and cut $\frac{4}{5}$ from that piece. What fraction of the full sheet do they use for this window?




$\frac{1}{2} \times \frac{4}{5} = \frac{4}{10}$ Four-tenths of the full sheet of glass was used.

118 Unit 3 • Lesson 3

Lesson 3
Problem-Solving Activity


Name _____ Date _____

3. At the next house, there are three broken windows. They cut $\frac{1}{3}$ of a full sheet of glass and then cut $\frac{1}{4}$ from one of the thirds. What fraction of the full sheet do they use for this window?




$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ One-twelfth of the full sheet of glass was used.

4. Next they take the other $\frac{2}{3}$ of the sheet and cut $\frac{3}{4}$ from that section to fix another window. What fraction of the full sheet do they use?




$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$ Six-twelfths of the full sheet of glass was used.

5. The repair crew cuts from another full sheet of glass for the last window. They cut $\frac{3}{4}$ of the full sheet and then cut $\frac{3}{4}$ from the $\frac{3}{4}$ sheet. What fraction of the full sheet do they use for this window?



$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$ Nine-sixteenths of the full sheet of glass was used.

 Reinforce Understanding
Use the Unit 3 Lesson 3 Problem Solving Teacher Talk Tutorial to review lesson concepts.

Unit 3 • Lesson 3 119

Student Workbook Pages with Answers

Lesson 32: Student Workbook

IRG 32

Problem Solving

Name _____

Date _____

Problem Solving Activity
Word Problems

Solve the following word problems. Use area models to help you solve each problem.

Haltom Glass Company replaces cracked and broken glass windows. When the repairers arrive at a house, they cut the glass for the windows from a large sheet of glass like the one shown. The repair crew has windows to fix at two different houses today.

- At the first house, the repair crew has to fix two windows. For one window, they cut the sheet of glass in half and then cut from one of the halves. What fraction of the full sheet do they use for this window?
- They take the other half of the full sheet of glass and cut from that piece. What fraction of the full sheet do they use for this window?

IRG • Lesson 32

3

IRG – 32

Problem Solving (cont.)

Name _____

Date _____

- At the next house, there are three broken windows. They cut $\frac{1}{3}$ of a full sheet of glass and then cut $\frac{1}{3}$ from one of the thirds. What fraction of the full sheet do they use for this window?
- Next they take the other $\frac{2}{3}$ of the sheet and cut $\frac{3}{8}$ from that section to fix another window. What fraction of the full sheet do they use?
- The repair crew cuts from another full sheet of glass for the last window. They cut $\frac{3}{4}$ of the full sheet and then cut $\frac{3}{4}$ from the $\frac{3}{4}$ sheet. What fraction of the full sheet do they use for this window?

IRG • Lesson 32

4

Lesson 40: Teacher Materials & Student Text

Problem Solving: ► Multiplication and Division Word Problems

How should we think about division to help us solve problems?
(Student Text, page 206)

- Have students turn to page 206 of the *Student Text*. Tell them that we have talked a lot about multiplication and division of fractions and now they will see some examples of how they are used in the real world. Tell them that it helps to remember to think about them in the ways we have discussed in the unit. For instance, remember that a fraction times a fraction may be thought of as a fraction of a fraction. Explain this language is very important when we solve word problems.
- Have students look at the problem $\frac{1}{3} \cdot \frac{3}{4}$. It helps to think of this problem as $\frac{1}{3}$ of $\frac{3}{4}$. Point out that this language helps us understand why the product is less than the factor. It's $\frac{1}{3}$ of the factor $\frac{3}{4}$.
- Discuss the language of division. It's important to think about how many groups of a fraction are found in another fraction. Have students look at the problem $\frac{3}{4} \div \frac{1}{4} = 3$. At first, it seems odd that the answer 3 is so large. Remind students that the division problem asks the question "How many groups of $\frac{1}{4}$ are there in $\frac{3}{4}$?" When it is phrased that way, the greater answer of 3 makes a lot more sense.

Lesson 9

► Problem Solving: Multiplication and Division Word Problems

How should we think about division to help us solve problems?

We have talked about the properties of multiplication and division of fractions throughout this unit. A way to think about multiplication of proper fractions is that we are taking a fraction of another fraction. This language helps us understand multiplication.

The problem $\frac{1}{3} \cdot \frac{3}{4}$ is the same as $\frac{1}{3}$ of $\frac{3}{4}$.

Because $\frac{1}{3}$ of $\frac{3}{4}$ is less than $\frac{3}{4}$, the product will be less than $\frac{3}{4}$.

$$\frac{1}{3} \cdot \frac{3}{4} = \frac{3}{12}$$

The product $\frac{3}{12}$ is less than $\frac{3}{4}$.

The language we use can also help us understand division. When we divide fractions, we talk about how to divide or break up one fraction by another.

$$\frac{3}{4} \div \frac{1}{4} = 3$$

In this case, we are using groups of $\frac{1}{4}$ to break up $\frac{3}{4}$.

How many groups of $\frac{1}{4}$ do we get when we divide $\frac{3}{4}$ by $\frac{1}{4}$?

We get 3 units. This helps us understand why the answer is 3. The number 3 tells us how many groups. It is not the size of the fraction.

In a bigger way, division is about two questions:

- How many are there in each group?
- How many groups are there?

The way we are dividing fractions answers the question, "How many groups do we have?" We will get a chance to think more about the properties and language of multiplication and division in today's problem-solving activity.

Problem-Solving Activity
Turn to Interactive Text,
page 139.

Reinforce Understanding
Use the Unit 2 Lesson 9 Problem Solving
Teacher Talk Tutorial to review lesson concepts.

206

- Tell students that it's helpful to remember with division that there are two main questions that the problem might be asking us to answer: "How many are there in each group?" or "How many groups are there?" Most of the problems we will look at are going to ask "How many groups are there?"
- Tell students that today's problem-solving activity will give them a chance to think more about the unique properties of multiplication and division with fractions and also the importance of the language that we use to describe them.



Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *Unit 3 Lesson 9 Problem Solving Teacher Talk Tutorial*.

Unit 3 • Lesson 9 331

Lesson 9

► Problem Solving: Multiplication and Division Word Problems

How should we think about division to help us solve problems?

We have talked about the properties of multiplication and division of fractions throughout this unit. A way to think about multiplication of proper fractions is that we are taking a fraction of another fraction. This language helps us understand multiplication.

The problem $\frac{1}{3} \cdot \frac{3}{4}$ is the same as $\frac{1}{3}$ of $\frac{3}{4}$.

Because $\frac{1}{3}$ of $\frac{3}{4}$ is less than $\frac{3}{4}$, the product will be less than $\frac{3}{4}$.

$$\frac{1}{3} \cdot \frac{3}{4} = \frac{3}{12}$$

The product $\frac{3}{12}$ is less than $\frac{3}{4}$.

The language we use can also help us understand division. When we divide fractions, we talk about how to divide or break up one fraction by another.

$$\frac{3}{4} \div \frac{1}{4} = 3$$

In this case, we are using groups of $\frac{1}{4}$ to break up $\frac{3}{4}$.

How many groups of $\frac{1}{4}$ do we get when we divide $\frac{3}{4}$ by $\frac{1}{4}$?

We get 3 units. This helps us understand why the answer is 3. The number 3 tells us how many groups. It is not the size of the fraction.

In a bigger way, division is about two questions:

- How many are there in each group?
- How many groups are there?

The way we are dividing fractions answers the question, "How many groups do we have?" We will get a chance to think more about the properties and language of multiplication and division in today's problem-solving activity.



Problem-Solving Activity
Turn to Interactive Text,
page 139.



Reinforce Understanding
Use the Unit 2 Lesson 9 Problem Solving
Teacher Talk Tutorial to review lesson concepts.

206 Unit 3 • Lesson 9

Lesson 40: Teacher Materials - Answer Key

Lesson 9


Problem-Solving Activity (Interactive Text, pages 139–140)

Have students turn to pages 139 and 140 of the *Interactive Text*, which provide an opportunity to practice multiplication and division with fractions. The context is fractional pounds of candy purchased in a candy store. Encourage students to use Cuisenaire rods to model each problem.

Monitor students' work as they complete these tasks. While circulating around the room, ask different students to explain their thinking.

Watch for:

- Can students recognize when a problem is asking for division rather than multiplication?
- Can students determine what the fractions are in the problem and know how to model them?
- Can students come up with the correct product or quotient that answers the problem?

 **Reinforce Understanding**
Remind students that they can review lesson concepts by accessing the online *Unit 3 Lesson 9 Problem Solving Teacher Talk Tutorial*.

Lesson 9 Problem-Solving Activity

Name _____ Date _____

Problem-Solving Activity
Multiplication and Division Word Problems

Candy Time is an old-fashioned candy store in the mall. Chocolates, licorice, jelly beans, taffy, and many other kinds of candy are sold in the store. Genna is the store owner, and she uses fractions in the store every day.

Use Cuisenaire rods or an area model to solve each problem. Draw a diagram that shows how the problem is solved.

1. Chocolate peanuts come in three-pound bags. Genna wants to put the chocolate peanuts in clear bags that weigh $\frac{1}{2}$ pound. How many clear bags of chocolate peanuts can Genna get from a three-pound bag?

$3 \div \frac{1}{2} = 6$; Genna can get six clear bags of chocolate peanuts.

2. Jelly beans are sold by the pound. Each little bin of jelly beans weighs $\frac{1}{3}$ pound. How much does $\frac{1}{2}$ bin of jelly beans weigh?

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$; A $\frac{1}{2}$ bin of jelly beans weighs $\frac{1}{6}$ pound.

3. The red licorice comes in long strips that are $\frac{3}{4}$ foot long. That's too long to sell. Genna needs to cut the strips into pieces that are $\frac{1}{4}$ foot long. How many $\frac{1}{4}$ -foot-long pieces can she get out of a $\frac{3}{4}$ -foot-long strip of licorice?

$\frac{3}{4} \div \frac{1}{4} = 3$; Genna can get three $\frac{1}{4}$ -foot-long pieces of licorice.

Unit 3 • Lesson 9 139

Lesson 9 Problem-Solving Activity

Name _____ Date _____

4. Milk chocolate is one of the most popular items in the store. Genna weighed a bag of milk chocolate and found each bag weighs $\frac{7}{8}$ pound. How much would $\frac{1}{2}$ of the bag weigh?

$\frac{1}{2} \times \frac{7}{8} = \frac{7}{16}$; One-half of the bag would weigh $\frac{7}{16}$ pound.

5. Genna puts the little pieces of white chocolate into bags that weigh $\frac{1}{8}$ of a pound. A customer bought 5 bags of white chocolate. How much does 5 bags weigh?

$5 \times \frac{1}{8} = \frac{5}{8}$; Five bags weigh $\frac{5}{8}$ pound.

6. Apple sours are also a big seller. One scoop of apple sours weighs $\frac{2}{3}$ pound. How much would $\frac{1}{4}$ scoop weigh?

$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$; One-quarter scoop weighs $\frac{1}{6}$ pound.

7. The caramels that Genna sells are expensive, so customers only buy two or three at a time. The caramels come in $\frac{3}{4}$ -pound bags. Genna puts three caramels in each clear bag. The clear bag of caramels weighs $\frac{1}{4}$ pound. How many clear bags of caramels can she get from the $\frac{3}{4}$ -pound bag?

$\frac{3}{4} \div \frac{1}{4} = 6$; Genna can get six clear bags of caramels.

8. Genna sells really hot fireballs. A scoop of fireballs weighs $\frac{3}{5}$ pound. How much would $\frac{1}{2}$ scoop weigh?

$\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$; One-half scoop weighs $\frac{3}{10}$ pound.

Reinforce Understanding
Use the Unit 3 Lesson 9 Problem Solving Teacher Talk Tutorial to review lesson concepts.

140 Unit 3 • Lesson 9

Student Workbook Pages with Answers

Lesson 40: Student Workbook

Name _____ Date _____

**Problem Solving Activity****Multiplication and Division Word Problems**

Candy Time is an old-fashioned candy store in the mall. Chocolates, licorice, jelly beans, taffy, and many other kinds of candy are sold in the store. Genna is the store owner, and she uses fractions in the store every day.



Use Cuisenaire rods or an area model to solve each problem. Draw a diagram that shows how the problem is solved.

1. Chocolate peanuts come in three-pound bags. Genna wants to put the chocolate peanuts in clear bags that weigh $\frac{1}{2}$ pound. How many clear bags of chocolate peanuts can Genna get from a three-pound bag?
2. Jelly beans are sold by the pound. Each little bin of jelly beans weighs $\frac{3}{4}$ pound. How much does $\frac{1}{2}$ bin of jelly beans weigh?
3. The red licorice comes in long strips that are 4 feet long. That's too long to sell. Genna needs to cut the strips into pieces that are $\frac{1}{4}$ foot long. How many $\frac{1}{4}$ foot-long pieces can she get out of the long strip of licorice?

Name _____ Date _____

4. Milk chocolate is one of the most popular items in the store. Genna weighed a bag of milk chocolate and found each bag weighs $\frac{7}{8}$ pound. How much would $\frac{1}{2}$ of the bag weigh?
5. Genna puts the little pieces of white chocolate into bags that weigh $\frac{1}{8}$ of a pound. A customer bought 5 bags of white chocolate. How much does 5 bags weigh?
6. Apple sours are also a big seller. One scoop of apple sours weighs $\frac{2}{3}$ pound. How much would $\frac{1}{4}$ scoop weigh?
7. Genna sells really hot fireballs. She has 2 customers who want to share a scoop. A scoop of fireballs weighs $\frac{1}{5}$ pound. How much does each customer get?

Practice Word Problem Solving

IRG-23 p.3 (explanation problem)

IRG-23 p. 4 one problem

IRG-32 p. 4 (3 problems; we will discuss #5)

IRG-49 p. 1 (3 problems)

Practice

Partner up

Prep and Practice

Lesson 13: Benchmarks and Relative Size

Lesson 27: Error Analysis

Lesson 39: Multiply and Commute

Lesson 43: Converting improper to mixed and word problems

Thoughts/Questions/Comments

Concluding Reminders

Handout

Concluding Reminders Handout

WRAP-UP

You'll get your official schedule with student names soon

We will monitor recordings more heavily in first two weeks so we can support any issues

Please reach out to Robin with content related issues

We will contact you if we determine there are any issues on your recordings

Questions?