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PREVENTING FAILURE IN FRACTIONS: ACCESSING GRADE-LEVEL FRACTIONS CONTENT THROUGH SMALL-GROUP INTERVENTION

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PREVENTING FAILURE IN FRACTIONS: ACCESSING GRADE-LEVEL FRACTIONS CONTENT THROUGH SMALL-GROUP INTERVENTION

OVERVIEW OF THE PROBLEM

A solid foundation in fractions is a key predictor of success in algebra and more advanced mathematics, which in turn is predictive of entry into higher education and many high-paying vocations. In fact, knowledge of fractions at age 10 (i.e., fifth grade) was found to predict performance in algebra above and beyond general math ability, IQ, or SES.[1] Yet, when fractions are introduced into the mathematics curriculum in upper elementary grades (grades 4 and 5), many students demonstrate minimal growth in fractions knowledge and some students find themselves three grade levels or more behind their peers at the start of seventh grade.[2, 3]

Fractions present increased challenges to students because they are more abstract than whole numbers. For example, fractions require an understanding of the relationship between the numerator and denominator to determine magnitude or value, whereas, whole number magnitude is more concrete and corresponds to counting discrete objects. As students are confronted with the abstract nature of fractions, many fall behind, which is likely to have long term consequences on their mathematics learning trajectory and even eliminate potential career options. Therefore, identifying students who struggle with fractions and preventing long term failure is critical.

Unfortunately, schools tend to focus on reading interventions to the exclusion of comparable interventions in mathematics.[4] And relatedly, there are fewer evidence-based resources in mathematics available to schools and teachers. Identifying the best way to meet the needs of students who struggle with fractions, coupled with validating instructional approaches that address this need, is critical.

This brief paper presents research findings from an experimental study utilizing a ***preventative*** approach to fraction intervention for fifth-grade students that emphasizes ***proactively*** teaching grade-level standards in conjunction with key foundations for understanding fractions concepts and procedures.[5, 6] Then, six recommendations for practice and policy are presented based on findings from the research study.

ADDRESSING THE PROBLEM THROUGH RESEARCH-BASED FRACTIONS INTERVENTION

A randomized controlled trial (RCT)[7] involving 186 students in 14 schools in 2 states, rigorously evaluated the impact of ATM (Adapted TransMath) Fractions Intervention on students struggling with fifth-grade-level fractions standards. The ATM Fractions Intervention is a small-group fractions intervention that incorporates current thinking in mathematics education, cognitive psychology, and special education. Students who participated in ATM Fractions Intervention performed significantly better than those who did not receive the intervention on a range of measures assessing fractions achievement, including knowledge and understanding of fifth-grade fractions concepts and procedures. A description of the study and results are below.

Student participants. Fifth-grade students who scored between the 15th and 37th percentile on a fourth-grade-level fractions screener participated in the study. This range of scores corresponds to answering 35–50% of the items correctly, which indicates that students were starting fifth grade with major gaps in foundational fourth-grade knowledge and were likely to struggle with fifth-grade material.

Instructional setting. The intervention was provided in small-groups, as a supplement to

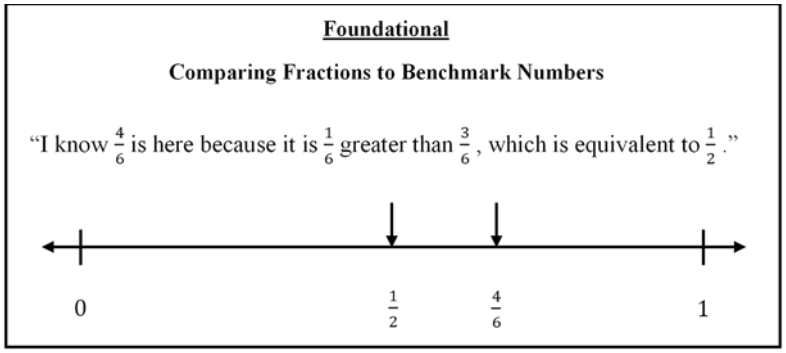
core mathematics. This model is in line with a Response to Intervention or Multi-Tiered Systems of Support framework where students who need additional support receive intervention. Many schools do not have such a framework in mathematics, even though providing reading intervention in this manner is commonplace.

ATM Fractions Intervention

Fractions content. Instruction focused on ***foundational*** and ***grade-level*** material through engaging and hands-on learning experiences. The ATM Fractions Intervention was adapted from the *TransMath*® curriculum[8] so that it could be implemented in small groups of 3-5 students for fifth-grade students in need of additional support in grade-level and foundational fractions material.[9]

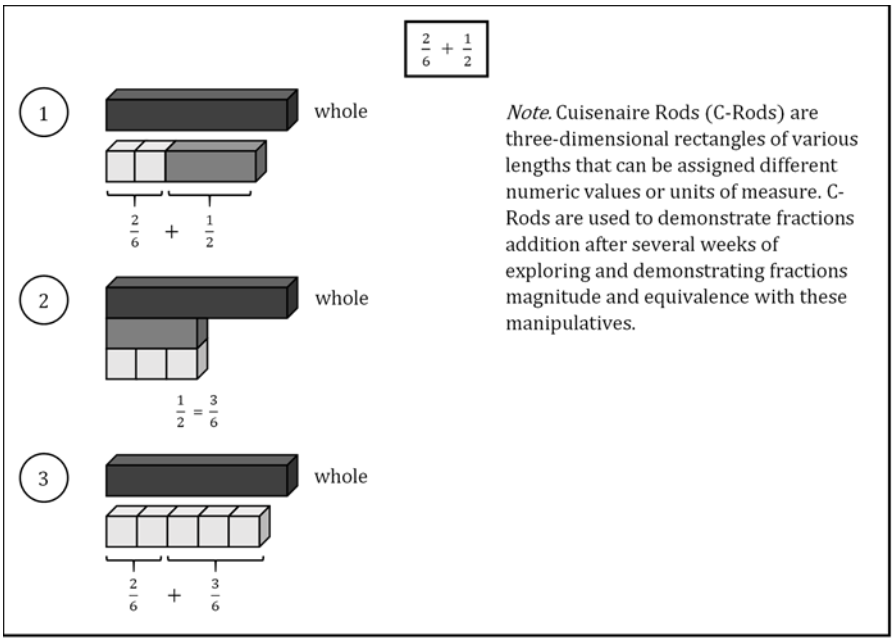
Initial lessons emphasized the ***foundational material*** most relevant to success in the fifth-grade standards, especially understanding fractions equivalence and fractions magnitude, both of which are included in the fourth-grade standards. ATM Fractions Intervention incorporates concrete representations that students can manipulate (Cuisenaire rods) and visual representations (number lines) to develop understanding of fractions magnitude (See Figure 1). In using these multiple representational tools, ATM Fractions Intervention intentionally addresses two types of fractions interpretations (a) the partitive (part-whole) interpretation of fractions, and (b) the more difficult, but essential, measurement interpretation of fractions.[10] Both of these interpretations are critical for moving into more advanced mathematics in middle school and are salient for understanding fifth-grade-level material.

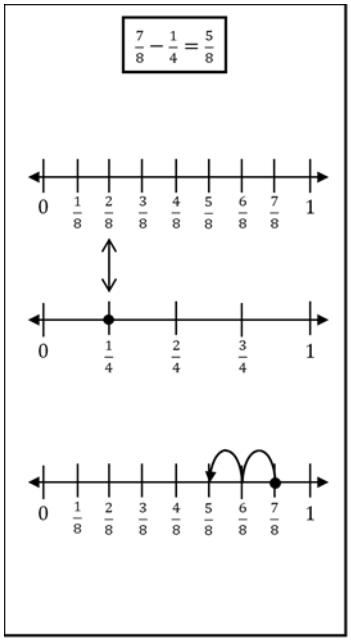
Figure 1. Estimating fractions magnitude on the number line.



Latter lessons systematically build on the foundational material to encompass ***grade-level material*** (i.e., fractions operations). Understanding fractions magnitude is essential to developing the ideas and principles necessary for understanding fractions operations (addition, subtraction, multiplication, and division) and for analyzing corresponding word problems as articulated in the CCSS-M.[11] These lessons also utilized concrete and visual representations to link concepts to fractions computation problems (see Figures 2 and 3). The ATM Fractions Intervention includes a strategic balance between conceptual understanding and procedural competence.[12, 13]

Figures 2 (left) and 3 (right). Linking concepts to procedures.





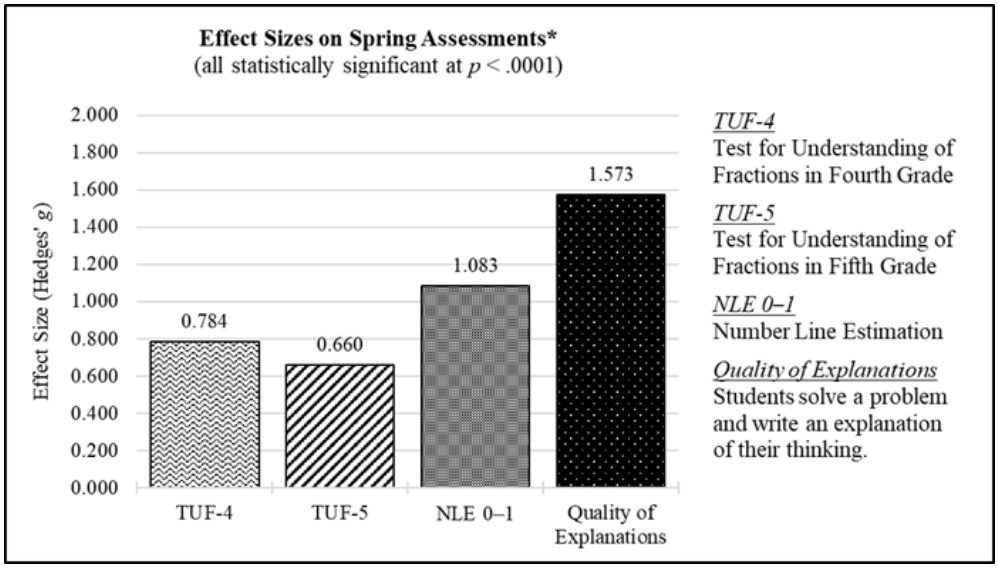
Evidence-based instructional practices to support struggling learners. The ATM Fractions Intervention incorporates evidence-based instructional approaches that appear in findings from RCTs related to this population of students.[14, 15, 16, 17, 18] These include the use of (a) multiple representations, (b) systematic instruction, (c) cumulative review and practice, and (d) providing immediate and frequent feedback to students. These practices have been shown time and again as efficacious for struggling learners.

In addition to these evidence-based practices, a critical element of the ATM Fractions Intervention was supporting students' in providing mathematically correct explanations of fractions concepts and procedures. Teachers first modeled explaining the concepts and procedures of problems through a "think-aloud" as they demonstrated problem solving. The expectation is that students would then move toward explaining their own thinking in this manner as they solved and modeled problems for their peers in the small group. Teachers also asked prompting and clarifying questions to elicit student thinking during their explanations to foster deeper insights, rather than providing students with answers.

Research Findings

Students who participated in the ATM Fractions Intervention significantly outperformed those who did not receive the intervention on all outcomes (Hedges g = .66 to 1.57, p<.0001). These effect sizes are considered moderate to large. See Figure 4.

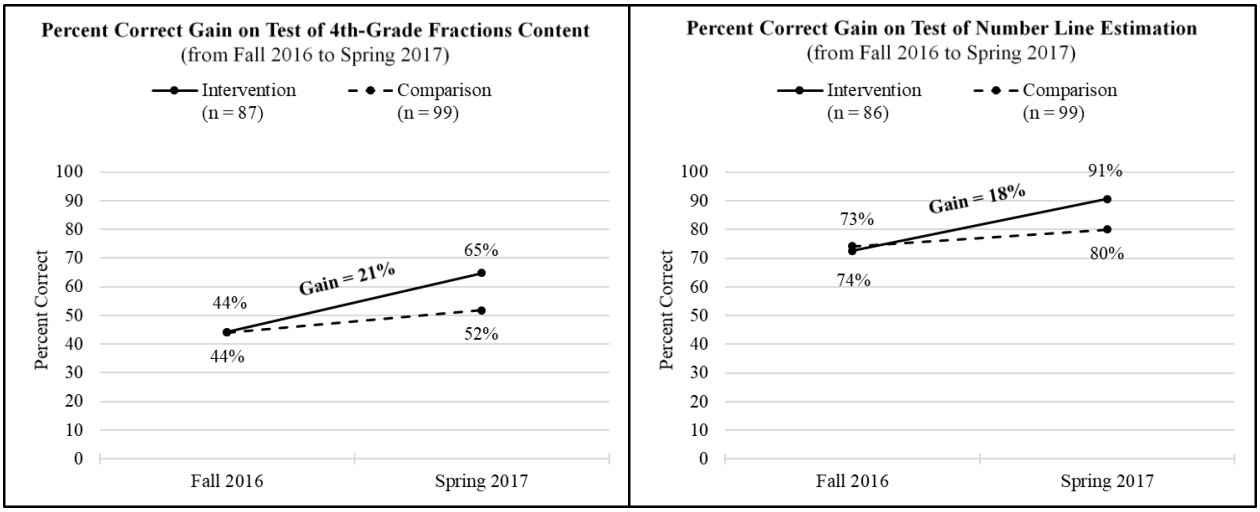
Figure 4. Effect sizes on spring assessments.



Note. Sample sizes for *TUF-4*: I = 87, C = 99; *TUF-5*: I = 86, C = 99; *NLE 0–1*: I = 86, C = 99; Quality of Explanations: I = 33, C = 34.

Students also demonstrated growth in fourth-grade foundational fractions understanding. See Figures 5 and 6. Both figures show the difference in the average growth trajectory between students who participated in intervention and those who were in the comparison group, that is, students who scored similarly on the screener but did not participate in the ATM Fractions Intervention.

Figures 5 (left) and 6 (right). Growth on fractions knowledge.



Note. Intervention = students who received fractions tutoring. Comparison = students who did not receive fractions tutoring.

IMPLICATIONS FOR POLICY AND PRACTICE

Increasing the number of students who can access and learn grade-level mathematics in upper elementary school, when standards include more challenging abstract fractions concepts, will require an investment on the part of policy makers and practitioners as they review their priorities in STEM learning. Six recommendations for practitioners and policy makers are presented below and are based on lessons learned from the study on ATM Fractions Intervention.

RECOMMENDATION 1: Use universal screening to identify students who are likely to struggle with fractions in upper elementary school.

Identifying students who are not well prepared for grade-level material is the first step in supporting students who are likely to fall behind in developing fraction understandings. Screeners that identify students who are weak in the prior year's content are most useful. Using either a fraction-specific or general mathematics screener is useful in identifying these students. Alternatively, the state assessment score from the previous spring may be a realistic and less burdensome option for schools.

RECOMMENDATION 2: Provide 25-35 minutes of mathematics intervention time as a supplement to core mathematics instruction for students who struggle.

Most schools provide reading intervention services within a tiered framework, often called Response to Intervention or Multi-Tiered Systems of Support. However, this provision of services is less often utilized in mathematics. Scheduling a 25- to 35-minute mathematics intervention time is recommended as a means to providing validated intervention to students who need it. Without an additional intervention time, these students will receive all of their mathematics instruction in their core mathematics classroom. In this setting, teachers are unlikely to have the time to provide these students with the additional instruction they need to access grade-level material.

RECOMMENDATION 3: Provide intervention in small groups consisting of 4-5 students.

One key feature in providing intervention is decreasing group size. In small groups, teachers are able to provide tailored, frequent, and immediate feedback to all students. Teachers can check student knowledge more easily in a small group and support students' explanations of key mathematical principles and ideas more often. A small-group environment allows for the pace of instruction to be slowed or for the material to be reviewed more often, depending on what students know or do not know.

RECOMMENDATION 4: Teach both foundational and grade-level fractions material during the intervention.

A remedial approach to students who struggle with grade-level material has been widely used in special education and in services provided by Title 1. Yet, if the goal of preventive intervention is to accelerate learning so students can access grade-level material, then including a

mix of the foundational material necessary for understanding grade-level concepts along with grade-level material will be beneficial for students.

RECOMMENDATION 5: Include evidence-based instructional practices that link fractions concepts to procedures.

Evidence-based instructional practices for students struggling with fractions include using multiple representations to link concepts and procedures, focusing on magnitude through estimating fractions magnitude on a number line, and supporting students in providing verbal and written explanations of mathematical principles and ideas. Each practice is described below.

Use ***multiple representation to teach fraction concepts.*** Multiple representations such as concrete materials (Cuisenaire Rods), visual representations, and mathematical notation are recommended as best practice for intervention in mathematics at any level. The advantage to these representations for fractions is that students can connect abstract concepts to procedures in a hands-on and visual manner.

Use ***the number line to teach fractions magnitude.*** Having a solid understanding of fractions magnitude (foundational material) is key to understanding grade-level fractions operations. The number line is considered the most mathematically accurate way to represent fractions and other rational numbers[19] and is a mainstay feature of fractions instruction in Asian countries who lead the world in STEM education.

Support students as they explain fractions concepts. Provide frequent opportunities for students to verbally explain their thinking when solving fractions operations and word problems.[20] Build in additional time during each intervention session for students to verbally explain their problem solving. Also provide opportunities for students to provide written explanations of their work.

RECOMMENDATION 6: Provide professional development to school personnel (teachers or paraprofessionals) providing the fractions intervention.

Professional development that accompanies any new program increases the likelihood that teachers will implement the new practice.[21] Professional development can focus on (a) representing fractions concepts and operations with multiple representations (Cuisenaire Rods and Number lines) in a mathematically precise fashion, (b) modeling think-alouds and using strategies to clarify and probe students' explanations, and (c) taking an in-depth look at the materials and lesson structure of the intervention.

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